Ramsey Taxation in the Global Economy*

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Abstract

We revisit classic issues of Ramsey taxation in the open economy: Should capital income be taxed? Should it be taxed along the transition? Should there be restrictions to free trade and capital mobility? Should goods be taxed based on origin or destination? What are desirable border adjustments? Should assets income be taxed based on source or residence? We characterize optimal wedges and analyze alternative policy implementations.

Keywords: Capital income tax; free trade; value added taxes; border adjustment; source and residence based taxation; origin and destination based taxation; production efficiency

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1 Introduction

We revisit the classic issues of Ramsey taxation in the open economy. Should capital income be taxed? Should it be taxed along the transition? Should there be restrictions to free trade and capital mobility? Should goods be taxed based on origin or destination? What are desirable border adjustments? Should assets income be taxed based on source or residence?

In this paper we take the Ramsey approach to optimal taxation, in that the tax system is exogenously given. We consider taxes widely used in practice in developed economies. Those include consumption and labor income taxes, taxes on dividends or equity returns, value added taxes with and without border adjustments, among others. We refer to such a tax system as a *rich tax system.* As is well known many tax policies yield the same distortions and the theory pins down those distortions in choices. Following the public finance literature we refer to these distortions as wedges.

The first question we address in this paper is what are the optimal wedges. In particular we ask whether the Ramsey policy yields intertemporal wedges. If it does, we say future capital is taxed. If it does not, we say future capital is not taxed. We also ask whether the Ramsey allocations distort conditions for production efficiency, associated with free trade. The second question is how the optimal wedges can be implemented. We consider implementations that, we believe, are of interest to policy design.

We consider a neoclassical growth model with two countries with intermediate goods that are traded internationally, as in Backus, Kehoe and Kydland (1994). We characterize the optimal cooperative Ramsey allocations and determine what are optimal intertemporal distortions and distortions on the movement of goods across borders. We determine what are the minimal set of fiscal instruments that implement those allocations and study alternative sets of instruments that implement those same allocations.

One of the main results is that for standard macro preferences, with constant elasticities, it is not optimal to impose intertemporal distortions, meaning that it is not optimal to tax capital. This result holds for the steady state but also for the transition. For more general preferences, capital is not taxed in the steady state, and there is no presumption that it ought to be taxed along the transition. A subsidy may be optimal. Another main result is that free trade, required for production efficiency, is also optimal.

A minimal set of instruments to implement the Ramsey allocation are consumption and labor income taxes. There is no need for taxes on income from assets.[1] For standard macro preferences only a constant tax on either labor or consumption is necessary to implement the Ramsey allocation.

We move on to consider alternative implementations where assets are taxed. We consider systems with profit taxes, taxes on dividends or equity returns and taxes on returns from foreign assets. We determine what are optimal policies and discuss issues of source versus residence based taxation.

We also consider alternative ways of taxing goods, in particular value added taxes with and without border adjustments. A tax system with value added taxes with border adjustments is equivalent to the system with consumption taxes. Instead, a value added tax without border adjustment in general would distort the allocation of capital across countries. A tariff, common across countries, but time varying could undo those distortions. We discuss the implications of these results for the desirability of origin versus destination based tax systems.

Our result that there is no presumption that capital ought to be taxed, not only in the steady state but also along the transition, is in contrast with influential results in the literature on the optimal taxation of capital. Chamley (1986) and Judd (1985) show that capital should be taxed at its maximum level initially and for a number of periods. Bassetto and Benhabib (2006) and Straub and Werning (2015) show that full taxation of capital can last forever.[2] This literature leads to the presumption that capital taxes should be high for a length of time. Two assumptions are important for the contrasting results.

The first assumption is that the (rich) tax system we consider is considerably less restrictive than the ones considered in that literature. While the literature on the optimal taxation of capital compares a tax on labor income to a tax on capital income restricted not to exceed 100%, we allow for other taxes that are commonly used in advanced economies. Once we allow for a less restricted tax system, the confiscation of initial wealth would justify at most a single intertemporal distortion in the initial period. For standard macro preferences, capital income should not be taxed from the second period one.

[1] This does not mean that capital is not taxed, since intertemporal distortions may be optimal along the transition for general preferences.

[2] Other relevant literature includes Chari, Christiano, Kehoe (1994), Atkeson, Chari and Kehoe (1999), Judd (1999, 2002), Coleman (2000), Lucas and Stokey (1983).

The second important assumption concerns the confiscation of initial wealth. A central feature of the literature on optimal taxation is that, absent other restrictions, factors in fixed supply should be taxed away completely. This feature implies that in the growth model, with a representative agent, the initial capital, as well as holdings of government bonds, should be taxed, possibly at rates in excess of 100% in order to fund government spending. In the Ramsey literature it is conventional to impose restrictions on such taxes on initial wealth. Given those restrictions, the planner will have an incentive to confiscate wealth indirectly through valuation effects. We depart from this literature by imposing a restriction on the value of initial wealth, rather than on the taxes themselves (see Armenter (2008) for an analysis with such a restriction). With this alternative restriction, the planner is limited on how much it can confiscate both directly and indirectly through valuation effects. For standard preferences and with such a restriction, future capital should never be taxed. With the conventional restriction in the Ramsey literature, capital accumulation is distorted in the very first period and is undistorted thereafter.

The conventional view of Ramsey equilibrium in dynamic economies is that the government chooses policies in period zero and commits to these policies thereafter. Our results hold under this conventional view. We go on to show that our results hold under an alternative view. In this alternative view, the government in each period has partial commitment in the sense that it can commit to one period returns on assets in utility terms. That is, the government in the following period is free to choose policies as it desires but must respect the previously committed return constraints.

The idea behind this notion of partial commitment begins by noting that returns on assets must satisfy intertemporal Euler equations on the equilibrium path. This notion of partial commitment requires that returns on assets must satisfy those conditions both on and off the equilibrium path. We show that the Markov equilibrium in this set up coincides with the commitment equilibrium with restrictions on the value of wealth. Thus partial commitment provides one rationalization for studying Ramsey equilibria with value of initial wealth restrictions.

We go on to study partial commitment to future taxes, rather than returns on assets. This analysis is motivated by the analysis of Ramsey problems with exogenously specified initial taxes. In this case Markov equilibria do not coincide with the commitment equilibrium. The reason is that commitment outcomes face a time inconsistency problem as in Lucas and Stokey (1983). In particular with single period debt, the government has a strong incentive to choose policies so as to reduce the value of inherited debt. These incentives imply that Markov equilibria do not coincide with the commitment equilibrium.

We briefly analyze an economy with heterogeneous agents and show that our representative agent results hold in such economies. An interesting feature of heterogeneous agent economies is that even if initial policies are unrestricted, the Ramsey equilibrium could distort intratemporal margins in order to achieve redistributive roles (see Werning, 2007).

Finally we relate our results to the ones on uniform commodity taxation (Atkinson and Stiglitz, 1972). Standard preferences used in the macroeconomic literature are separable and homothetic in consumption and labor. With these preferences, the growth model can be recast as a model in which constant returns to scale technologies are used by competitive firms to produce final consumption and labor aggregates. In this recast economy, we show that the Diamond and Mirrlees (1971) production efficiency theorem can be extended to obtain that it is optimal not to distort the use of intermediate goods. These intermediate goods consist of consumption, labor and capital at each date in the original economy. This result implies that in the original economy, future capital should never be taxed.

The paper is organized as follows: We present the two country economy model with consumption and labor income taxes in Section 2. We compute optimal Ramsey allocations, show that trade should not be restricted and that, for standard macro preferences, capital should never be taxed. In Section 3, we consider alternative tax systems that implement the same Ramsey optimal allocation. We first consider standard asset income taxes, allowing for different treatment of foreign and domestic income (Section 3.1). We then consider a common tax on income from domestic equity and from foreign assets, together with a profit tax (Section 3.2). We discuss source versus residence based taxes. We also discuss alternative ways of taxing consumption through value added taxes with and without border adjustment (Sections 3.3 and 3.4). We move on to discuss the assumptions on the initial confiscation of capital that are key for the results on the taxation of capital (Section 4). We do this in the closed economy. We justify the assumptions on initial confiscation, by considering an equilibrium with one period commitment to returns on assets. The Ramsey solution in that model with one period commitment coincides with the full commitment solution with restricted confiscation. In Section 5 consider within country heterogeneity. For standard preferences tax on capital is zero regardless of the ownership of capital. Finally, in Section 6, we relate the results on the taxation of capital to production efficiency.

2 A two country economy

There are two countries in this economy indexed by $i = 1, 2$. The preferences of a representative household in each country are over consumption c_{it} and labor n_{it},

$$U^i = \sum_{t=0}^{\infty} \beta^t u^i \left(c_{it}, n_{it}\right), \tag{1}$$

satisfying the usual properties.

Following Backus, Kehoe and Kydland (1994) only intermediate goods are traded. Final goods are not traded.

Each country, $i = 1, 2$, produces a country specific intermediate good, y_{it}, according to a production technology given by

$$y_{i1t} + y_{i2t} = y_{it} = F^i \left(k_{it}, n_{it}\right) \tag{2}$$

where y_{ijt} denotes the quantity of intermediate goods produced in country i and used in country $j = 1, 2$, k_{it} is the capital stock, n_{it} is labor input and F^i is constant returns to scale. The intermediate goods produced by each country are used to produce a country specific final good that can be used for private consumption, c_{it}, public consumption, g_{it}, and investment, x_{it}, according to

$$c_{it} + g_{it} + x_{it} \leq G^i \left(y_{1it}, y_{2it}\right) \tag{3}$$

where G^i is constant returns to scale. Capital accumulates according to the law of motion

$$x_{it} = k_{it+1} - (1 - \delta) k_{it}. \tag{4}$$

If lump sum taxes and transfers across countries are available, the allocations on the Pareto frontier satisfy the following efficiency conditions,

$$-\frac{u^i_{ct}}{u^i_{nt}} = \frac{1}{G^i_{i,t} F^i_{nt}},\ i = 1, 2 \tag{5}$$

$$\frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = 1 - \delta + G^i_{i,t+1} F^i_{kt+1},\ i = 1, 2 \tag{6}$$

$$\frac{G^1_{j,t}}{G^1_{j,t+1}} \left[G^1_{1,t+1} F^1_{k,t+1} + 1 - \delta \right] = \frac{G^2_{j,t}}{G^2_{j,t+1}} \left[G^2_{2,t+1} F^2_{k,t+1} + 1 - \delta \right],\ j = 1, 2 \tag{7}$$

$$\frac{G^1_{1,t}}{G^2_{1,t}} = \frac{G^1_{2,t}}{G^2_{2,t}} \tag{8}$$

which, together with the resource constraints, characterize the Pareto frontier.

The conditions above mean that there are no intratemporal wedges (conditions (5)), no intertemporal wedges ((conditions (7)), and no production distortions (conditions (6) and (8)). Conditions (5) set the marginal rate of substitution between consumption and labor equal to the marginal productivity in each country. Conditions (6) equate the intertemporal marginal rate of substitution to the marginal productivity of capital. Conditions (7) equate the marginal rates of transformation of the same intermediate good in the two countries and conditions (8) equate the marginal rates of technical substitution for the two intermediate goods.

We can use the intratemporal and intertemporal conditions, (5) and (6), to write the intertemporal condition for labor,

$$\frac{u^i_{nt}}{\beta u^i_{n,t+1}} = \frac{G^i_{i,t} F^i_{nt}}{G^i_{i,t+1} F^i_{n,t+1}} \left[1 - \delta + G^i_{i,t+1} F^i_{kt+1} \right],\ i = 1, 2. \tag{9}$$

We explicitly characterize this intertemporal labor margin because are interested in understanding when it is optimal not to distort this margin.

Next we consider an economy with distorting taxes. Throughout we allow for transfers across governments. We begin by considering only country specific consumption and labor income taxes. We show that a cooperative Ramsey allocation has no intertemporal distortions (for standard macro preferences) and has production efficiency. We then include a richer tax system with alternative taxes and discuss alternative implementations. Finally we discuss what are important assumptions for the results concerning the initial confiscation of assets. We end by relating the absence of intertemporal distortions to production efficiency.

2.1 Competitive equilibria with consumption and labor income taxes

We now describe a competitive equilibrium with taxes in which governments finance public consumption and initial debt with (possibly) time varying proportional taxes on consumption and labor income, τ_{it}^{c} and τ_{it}^{n}, as well as a tax on initial wealth, l_{i0}.

Each country has two types of firms. Given that the technologies are constant returns to scale, we assume, without loss of generality that there are two types of representative firms. The *intermediate good firm* in each country uses the technology in (2) to produce the intermediate good using capital and labor, purchases investment goods, and accumulates capital according to (4).[3] Let V_{i0} be the value of the firm in period zero after the dividend paid in that period, d_{i0}. The intermediate good firm maximizes the value of dividends

$$V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t \left[p_{it} \left(y_{i1t} + y_{i2t} \right) - w_{it} n_{it} - q_{it} x_{it} \right] \tag{10}$$

subject to (2) and (4). where p_{it} is the price of the intermediate good in units of a numeraire (or common money across countries) at t, w_{it} is the wage rate, and q_{it} is the price of investment, or equivalently of the final good, all in units of the same numeraire. Q_t is the intertemporal price of the common numeraire at time t in units of the numeraire at zero ($Q_0 = 1$). Because the intermediate goods are traded, and there are no tariffs, the prices of each of the intermediate goods are the same in the two countries.

If we define r_{t+1}^{f} to be the return on one period bonds in units of the numeraire between period t and $t+1$, then it must be the case that

$$\frac{Q_t}{Q_{t+1}} = 1 + r_{t+1}^{f}, \text{ for } t \geq 0. \tag{11a}$$

The *final goods firm* in each country uses the technology in (3) to produce the final good using foreign and domestically produced intermediate goods to maximize

[3] In Appendix 1 we describe, for a closed economy model, an alternative, more widely used decentralization in which the household owns the capital stock and firms rent capital from the household. The two decentralizations are equivalent, but it is easier to relate the taxes in the decentralization described here to the ones in existing tax systems.

the value of dividends

$$\sum_{t=0}^{\infty} Q_t \left[q_{it} G^i \left(y_{1it}, y_{2it} \right) - p_{1t} y_{1it} - p_{2t} y_{2it} \right]. \tag{12}$$

This problem reduces to a sequence of static problems.

Household The household budget constraint in each country is

$$\sum_{t=0}^{\infty} Q_t \left[q_{it} \left(1 + \tau_{it}^c\right) c_{it} - \left(1 - \tau_{it}^n\right) w_{it} n_{it} \right] \leq \left(1 - l_{i0}\right) a_{i0}, \tag{13}$$

with

$$a_{i0} = V_{i0} + d_{i0} + Q_{-1} b_{i0} + \left(1 + r_0^f\right) f_{i0},$$

where a_{i0} denotes net holdings of assets by the household of country i, $Q_{-1} b_{i0}$ denotes holdings of domestic public debt in units of the numeraire, inclusive of interest, $\left(1 + r_0^f\right) f_{i0}$ denotes holdings of claims on households in the other country, in units of the numeraire, also inclusive of interest. We assume that households within a country hold claims to all the firms in that country as well as the public debt of the government of that country. The households hold claims on the households in the other country.

The household's problem is to maximize utility (1), subject to (13).

Government The budget constraint of the government of each country is given by

$$\sum_{t=0}^{\infty} Q_t \left[\tau_{it}^c q_{it} c_{it} + \tau_{it}^n w_{it} n_{it} - q_{it} g_{it} \right] + l_{i0} a_{i0} = Q_{-1} b_{i0} - T_{i0}. \tag{14}$$

where T_{i0} received by the government of country i, from other governments, so that

$$T_{10} + T_{20} = 0. \tag{15}$$

The budget constraints of the government and the household (with equality) in each country imply

$$\sum_{t=0}^{\infty} Q_t \left[p_{it} y_{it} - q_{it} \left(c_{it} + g_{it} + x_{it} \right) \right] = -\left(1 + r_0^f\right) f_{i0} - T_{i0}, \tag{16}$$

which can be written as the balance of payments condition

$$\sum_{t=0}^{\infty} Q_t \left[p_{it} y_{ijt} - p_{jt} y_{jit}\right] = -\left(1 + r_0^f\right) f_{i,0} - T_{i0}. \tag{17}$$

with $\left(1 + r_0^f\right) f_{1,0} + \left(1 + r_0^f\right) f_{2,0} = 0.$

A *competitive equilibrium* for this economy consists of a set of allocations $\{c_{it}, n_{it}, y_{ijt}, k_{it+1}, x_{it}\}$ and d_{i0}, prices $\{q_{it}, p_{it}, w_{it}, Q_t, V_{i0}\}$, and policies $\{\tau_{it}^c, \tau_{it}^n, l_{i0}, T_{i0}\}$, given $\left\{k_{i0}, Q_{-1} b_{i0}, \left(1 + r_0^f\right) f_{i0}\right\}$ such that households maximize utility subject to their constraints, firms maximize value, the balance of payments conditions (17) hold, and markets clear in that (2), (3), and (4) together with (15) are satisfied.

Note that we have not explicitly specified the governments' budget constraints because they are implied by the other constraints.

We say that an allocation $\{c_{it}, n_{it}, y_{ijt}, k_{it+1}, x_{it}\}$ is *implementable* if it is part of a competitive equilibrium.

The first order conditions of the household's problem include

$$-\frac{u_{c,t}^i}{u_{n,t}^i} = \frac{\left(1 + \tau_{it}^c\right) q_{it}}{\left(1 - \tau_{it}^n\right) w_{it}}, \tag{18}$$

$$\frac{u_{c,t}^i}{\left(1 + \tau_{it}^c\right)} = \frac{Q_t q_{it}}{Q_{t+1} q_{it+1}} \frac{\beta u_{c,t+1}^i}{\left(1 + \tau_{it+1}^c\right)}, \tag{19}$$

for all $t \geq 0$, where $u_{c,t}^i$ and $u_{n,t}^i$ denote the marginal utilities of consumption and labor in period t.

The first order conditions of the firms' problems are, for all $t \geq 0$,

$$p_{it} F_{n,t}^i = w_{it} \tag{20}$$

$$Q_t q_{it} = Q_{t+1} p_{it+1} F_{k,t+1}^i + Q_{t+1} q_{it+1} \left(1 - \delta\right)) \tag{21}$$

where $F_{n,t}^i$ and $F_{k,t}^i$ denote the marginal products of capital and labor in period t, together with

$$q_{it} G_{j,t}^i = p_{jt}. \tag{22}$$

By combining household and firms equilibrium conditions it can be shown that the value of the firm in (10) is

$$V_{i0} + d_{i0} = q_{i0} \left[1 - \delta + G^i_{i,0} F^i_{k,0} \right] k_{i0}. \tag{23}$$

The first order conditions can be rearranged as

$$-\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{(1 + \tau^c_{it})}{(1 - \tau^n_{it}) G^i_{i,t} F^i_{n,t}}. \tag{24}$$

$$\frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = \frac{(1 + \tau^c_{it})}{\left(1 + \tau^c_{it+1}\right)} \left[G^i_{i,t+1} F^i_{k,t+1} + 1 - \delta \right] \tag{25}$$

as well as (7) and (8), repeated here,

$$\frac{G^1_{j,t}}{G^1_{j,t+1}} \left[G^1_{1,t+1} F^1_{k,t+1} + 1 - \delta \right] = \frac{G^2_{j,t}}{G^2_{j,t+1}} \left[G^2_{2,t+1} F^2_{k,t+1} + 1 - \delta \right], \; j = 1, 2$$

$$\frac{G^1_{2,t}}{G^1_{1,t}} = \frac{G^2_{2,t}}{G^2_{1,t}}$$

for all $t \geq 0$.

Comparing these conditions with the ones for the Pareto frontier with lump sum taxation, (5), (6), (7), and (8), we have that the consumption and labor taxes create an intratemporal wedge in (24), and that time varying consumption taxes create intertemporal wedges in (25). Taxes do not affect the production efficiency conditions (7) and (8).

Using conditions (24) and (25), we can write

$$\frac{u^i_{n,t}}{\beta u^i_{n,t+1}} = \frac{(1 - \tau^n_{it})}{\left(1 - \tau^n_{it+1}\right)} \frac{G^i_{i,t} F^i_{n,t}}{G^i_{i,t+1} F^i_{n,t+1}} \left[G^i_{i,t+1} F^i_{k,t+1} + 1 - \delta) \right] \tag{26}$$

which makes it clear how the taxes affect the labor intertemporal margin.

A competitive equilibrium has *no intertemporal distortions in consumption* from period s onwards if the first order conditions (25) and (6) coincide for all $t \geq s$. Similarly, a competitive equilibrium has *no intertemporal distortions in labor* from period s onwards if the first order conditions (26) and (9) coincide for all $t \geq s$. Finally, a competitive equilibrium has *no intertemporal distortions* from period s onwards if it has no such distortions for both consumption and labor.

With the taxes that we consider here, it is not possible to create production distortions in the use of the traded goods, so that (7) and (8) have to be satisfied always. These marginal conditions will have to be imposed as restrictions to the Ramsey problem, but as we show below, they will not be binding at the Ramsey optimum.

2.1.1 Implementability

In order to characterize the Ramsey equilibrium we begin by characterizing the set of implementable allocations. In the appendix we show that an allocation $\{c_{it}, n_{it}, y_{ijt}, k_{it+1}, x_{it}\}$ and period zero policies and prices, $\{l_{i0}, \tau_{i0}^c, T_{i0}, q_{i0}\}$, given $\{k_{i0}, b_{i0}, f_{i0}\}$ is implementable as a competitive equilibrium if and only if they satisfy the resource constraints (2), (3), (4), and the implementability conditions

$$\sum_{t=0}^{\infty} \left[\beta^t u_{c,t}^i c_{it} + \beta^t u_{n,t}^i n_{it}\right] = \mathcal{W}_{i0}, \tag{27}$$

where

$$\mathcal{W}_{i0} = (1 - l_{i0}) \frac{u_{c,0}^i}{(1+\tau_{i0}^c)} \left[\left(1 - \delta + G_{i,0}^i F_{k,0}^i\right) k_{i0} + Q_{-1} b_{i0} + \left(1 + r_0^f\right) \frac{f_{i,0}}{q_{i,0}}\right] \tag{28}$$

together with (7) and (8). We formally state the following proposition proved in the appendix

Proposition 1: (Characterization of the implementable allocations). Any implementable allocation and period zero policies and prices satisfies the implementability constraints (27), together with (7) and (8), as well as the resource constraints (2), (3), (4). Furthermore, if an allocation satisfies these conditions for some period zero policies and prices, then it is implementable by a tax system with consumption and labor income taxes.

2.2 Cooperative Ramsey equilibria

A (*Cooperative*) *Ramsey equilibrium* is a competitive equilibrium that is not Pareto dominated by any other competitive equilibrium. The *Ramsey allocation* is the associated implementable allocation.

We say that the Ramsey planner is unrestricted if the planner can choose policies and allocations in all periods subject only to the constraint that the resulting allocations, prices and policies constitute a competitive equilibrium.

Consider the following programming problem, referred to as the *Unrestricted Ramsey problem* which is to choose allocations and period zero policies to maximize a weighted sum of utilities of the households of the two countries,

$$\theta^1 U^1 + \theta^2 U^2 \tag{29}$$

with weights $\theta^i \in [0, 1]$, subject to the conditions (27) and the resource constraints.

Assume policies are unrestricted in the sense that for any allocation, l_{i0} (or any of the other initial taxes) can be chosen to satisfy (27). Then the unrestricted Ramsey problem reduces to maximizing welfare subject to the resource constraints, and therefore it immediately follows that it is possible to implement the lump-sum tax allocation as the Ramsey equilibrium.

2.2.1 Ramsey problem

Suppose now that policies and initial conditions are restricted in the sense that households in each country must be allowed to keep an exogenous value of initial wealth $\bar{\mathcal{W}}_i$, measured in units of utility. Specifically, we impose the following restriction on the Ramsey problem

$$\mathcal{W}_{i0} = \bar{\mathcal{W}}_i, \tag{30}$$

which we refer to as the *wealth restriction in utility terms*. One example of such a restriction with positive wealth is that tax on the initial wealth cannot exceed 100%. Then, since it is possible to set the tax on wealth equal to 100%, then $\bar{\mathcal{W}}_i$ is zero for $i = 1, 2$.

With this restriction, policies, including initial policies, can be chosen arbitrarily but the household must receive a value of initial wealth in utility terms of $\bar{\mathcal{W}}_i$ (see Armenter (2007) for an analysis with such a restriction). We show below that this outcome is the equilibrium outcome for an environment with partial commitment.

The Ramsey problem is to maximize (29), subject to the resource constraints (2), together with (3) and (4), that are combined as

$$c_{it} + g_{it} + k_{it+1} - (1 - \delta) k_{it} \leq G^1 (y_{1it}, y_{2it}) \tag{31}$$

together with the implementability conditions (27), the wealth restriction (30), (7) and

(8) . Condition (28) does not restrict the problem since it is satisfied by the choices of the initial taxes. We are going to write the problem without imposing the conditions for production efficiency, (7) and (8). We will show that they are satisfied at the optimum.

Define

$$v^i\left(c_{it}, n_{it}; \varphi^i\right) = \theta u^i\left(c_{it}, n_{it}\right) + \varphi^i\left[u^i_{c,t}c_{it} + u^i_{n,t}n_{it}\right]$$

where φ^i is the multiplier of the implementability condition (27). We can now use the efficiency conditions for the case with lump sum taxes, (5), (6), (7) and (8), replacing the marginal utilities of u^i by the derivatives of the function v^i. The solution of the Ramsey problem is given by

$$-\frac{v^i_{c,t}}{v^i_{n,t}} = \frac{1}{G^i_{i,t}F^i_{n,t}},\ i = 1, 2 \tag{32}$$

$$\frac{v^i_{c,t}}{\beta v^i_{c,t+1}} = 1 - \delta + G^i_{i,t+1}F^i_{kt+1},\ i = 1, 2 \tag{33}$$

$$\frac{G^1_{j,t}}{G^1_{j,t+1}}\left[G^1_{1,t+1}F^1_{k,t+1} + 1 - \delta\right] = \frac{G^2_{j,t}}{G^2_{j,t+1}}\left[G^2_{2,t+1}F^2_{k,t+1} + 1 - \delta\right],\ j = 1, 2 \tag{34}$$

$$\frac{G^1_{1,t}}{G^2_{1,t}} = \frac{G^1_{2,t}}{G^2_{2,t}} \tag{35}$$

Every Ramsey solution must satisfy the production efficiency conditions, (7) and (8), even if the conditions were not imposed as a restriction to the problem. This means that if we had considered tariffs as possible instruments, they would not need to be used. The proposition follows:

Proposition 2 (Optimality of free trade): Unrestricted international trade is optimal.

Sketch of proof: Consider a tax system consisting of taxes only on consumption and labor income in both countries and no tariffs are levied on goods as they cross borders. Notice that if the consumption and labor taxes are set at the levels associated with the Ramsey solution, then the associated competitive equilibrium implements the Ramsey outcome.

In order to further characterize the optimal wedges, it is useful to write

$$v^i_{c,t} = u^i_{c,t}\left[\theta^i + \varphi^i\left[1 - \sigma^i_t - \sigma^{cni}_t\right]\right]$$

$$v^i_{n,t} = u^i_{n,t}\left[\theta^i + \varphi^i\left[1 + \sigma^{ni}_t - \sigma^{nci}_t\right]\right]$$

where

$$\sigma^i_t = -\frac{u^i_{cc,t}c_{it}}{u^i_{c,t}},\ \sigma^{ni}_t = \frac{u^i_{nn,t}n_{it}}{u^i_{n,t}},\ \sigma^{nci}_t = -\frac{u^i_{nc,t}c_{it}}{u^i_{n,t}},\ \sigma^{cni}_t = -\frac{u^i_{cn,t}n_{it}}{u^i_{c,t}},$$

are own and cross elasticities, that are only functions of consumption and labor at time t.

Note also that if consumption and labor are constant over time, then the relevant elasticities are also constant so $v^i_{c,t}$ and $v^i_{n,t}$ are proportional to $u^i_{c,t}$ and $u^i_{n,t}$, respectively. It then follows that it is optimal to have no intertemporal distortions. This observation leads to the following proposition.

Proposition 3: (No intertemporal distortions in the steady state) If the Ramsey equilibrium converges to a steady state, it is optimal to have no intertemporal distortions asymptotically.

For standard macro preferences,

$$U^i = \sum_{t=0}^{\infty}\beta^t\left[\frac{c_t^{1-\sigma^i}-1}{1-\sigma^i} - \eta_i\frac{n_t^{1+\sigma^{ni}}}{1+\sigma^{ni}}\right], \tag{36}$$

the marginal conditions are

$$-\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{\theta^i + \varphi^i\left(1+\sigma^{ni}\right)}{\theta^i + \varphi^i\left(1-\sigma^i\right)}\frac{1}{G^i_{i,t}F^i_{n,t}} \tag{37}$$

together with the intertemporal efficiency conditions (6), and the production efficiency conditions, (7) and (8) The proposition follows:

Proposition 4: (No intertemporal distortions ever) Suppose that preferences are given by (36). Then, the Ramsey solution has no intertemporal distortions for all $t \geq 0$.

Corollary: The Ramsey allocations can be implemented with consumption or labor taxes that are constant over time, but possibly different across countries.

Note that the preferences above are separable and homothetic in both consumption and labor. We use these properties to provide intuition for the results in Section 6 below, where we relate them to results on uniform commodity taxation and production efficiency.

The Ramsey allocation characterized in propositions 2 through 4 can be implemented in a variety of ways. In any implementation, the initial wealth taxes, l_{i0}, are chosen to satisfy the wealth constraints.

3 Alternative implementations

In this section, we discuss a variety of other tax systems, including taxes on the income from different assets. Our analysis is motivated by the observation that these alternative tax systems are widely used in practice. We show that no tax system can yield higher welfare than the tax system with only consumption and labor income taxes. We show that a variety of tax systems can implement the Ramsey allocation associated with those taxes. Furthermore some tax systems do yield lower welfare.

We start with a tax system with taxes on the income from foreign assets and from dividends. There is also a capital income tax levied on firms profits.

3.1 Taxes on capital income, dividends, and foreign assets

We now consider a tax system that includes, in addition to consumption and labor income taxes, taxes on capital income τ_{it}^{k}, dividends τ_{it}^{d}, and taxes on income from net foreign assets τ_{it}^{f}. The purpose of this exercise is to relate the Ramsey intertemporal distortions derived above to the taxation of capital.

Capital accumulation is conducted by the (intermediate good) firms, and the firms pay the capital income taxes. We assume that the firms are owned by the domestic households, that trade shares and receive dividends. We now describe the problems of the firms and the household in each country and define a competitive equilibrium.

Firm The representative firm produces and invests in order to maximize the present value of dividends, net of taxes, $V_{i0} + \left(1 - \tau_{i0}^{d}\right) d_{i0} = \sum_{t=0}^{\infty} Q_{it} \left(1 - \tau_{it}^{d}\right) d_{it}$. Dividends, in units of the numeraire, d_{it}, are given by

$$d_{it} = p_{it}F(k_{it}, n_{it}) - w_{it}n_{it} - \tau_{it}^{k} \left[p_{it}F(k_{it}, n_{it}) - w_{it}n_{it} - q_{it}\delta k_{it}\right] - q_{it} \left[k_{it+1} - (1-\delta)k_{it}\right] \tag{38}$$

where τ_{it}^{k} is the tax rate on capital income net of depreciation. Notice that here the intertemporal prices of the numeraire Q_{it} are indexed by the country, because the agents in the two countries face different taxes. It has to be the case that $Q_{i0} = 1$, in both countries.

Note that in this way of setting up the competitive equilibrium, dividends are net payments to claimants of the firm. These payments could be interpreted either as payments on debt or as payments to equity holders. To clarify this interpretation, consider an all-equity firm. In this case, our notion of dividends consists of cash dividends plus stock buybacks less issues of new equity. In particular, under this interpretation taxes on capital gains associated with stock buybacks are assumed to be levied on accrual and at the same rate as cash dividends. Note also that dividends could be negative if returns to capital are smaller than investment. In this case, a positive tax on dividends would represent a subsidy to the firm. (In a steady state of the competitive equilibrium it is possible to show that dividends are always positive).

The first order conditions of the firm's problem are now (20) together with

$$\frac{Q_{it}q_{it}}{Q_{it+1}q_{it+1}} = \frac{\left(1-\tau_{it+1}^{d}\right)}{1-\tau_{it}^{d}}\left[1+\left(1-\tau_{it+1}^{k}\right)\left(\frac{p_{it+1}}{q_{it+1}}F_{k,t+1}^{i}-\delta\right)\right] \tag{39}$$

Substituting for d_{it} from (38) and using (20) and (39) it is easy to show that the after tax value of the dividends at time zero in units of the numeraire is given by

$$\sum_{t=0}^{\infty} Q_{it}\left(1-\tau_{it}^{d}\right)d_{it} = \left(1-\tau_{i0}^{d}\right)\left[1+\left(1-\tau_{i0}^{k}\right)\left(\frac{p_{i0}}{q_{i0}}F_{ik,0}-\delta\right)\right]p_{i0}k_{i0}. \tag{40}$$

The problem of the final good firm in each country is not affected by the capital income and dividend taxes. The first order condition is equation (22).

Households The flow of funds constraint in period t for the household in country i in units of the numeraire is given by

$$\begin{aligned} & b_{it+1}+V_{it}s_{it+1}+f_{it+1} \\ = \; & \frac{Q_{it-1}}{Q_{it}}b_{it}+V_{it}s_{it}+\left[1+r_{t}^{f}-\tau_{it}^{f}\left(r_{t}^{f}-\left(\frac{q_{it}}{q_{it-1}}-1\right)\right)\right]f_{it}+ \\ & \left(1-\tau_{it}^{d}\right)d_{it}s_{it}+\left(1-\tau_{it}^{n}\right)w_{it}n_{it}-\left(1+\tau_{it}^{c}\right)q_{it}c_{it}. \end{aligned} \tag{41a}$$

Taxation of the foreign assets allows for an inflation deduction. In period 0, the constraint is

$$
\begin{aligned}
& b_{i1} + V_{i0} s_{i1} + f_{i1} \\
= \; & (1 - l_{i0}) \left[Q_{i-1} b_{i0} + V_{i0} s_{i0} + (1 - \tau^d_{i0}) d_{i0} s_{i0} + \left[1 + r^f_0 - \tau^f_{i0} \left(1 + r^f_0 - \frac{q_{i0}}{q_{i-1}} \right) \right] f_{i0} \right] \\
& + (1 - \tau^n_{i0}) \, w_{i0} n_{i0} - (1 + \tau^c_{i0}) \, q_{i0} c_{i0}. \qquad (42)
\end{aligned}
$$

V_{it} is the (after dividend) value of firm's shares in units of the period t numeraire.[4] Notice that for the country where the foreign assets are negative, a positive τ^f_{it} represents a deductibility of interest payments. The interest rates on the foreign assets in units of the numeraire at time t, r^f_t, are not indexed by the country, since there must be a single price in that market.

The household's problem is to maximize utility (1) , subject to (41a), (42) and no-Ponzi scheme conditions, $\lim_{T\to\infty} Q_{iT+1} b_{iT+1} \geq 0$, and $\lim_{T\to\infty} Q_{iT+1} f_{iT+1} \geq 0$.

The first order conditions of the household's problem in each country are, for $t \geq 0$, (18), (19,)together with

$$
\frac{Q_{it}}{Q_{it+1}} = 1 + r^f_{t+1} - \tau^f_{it+1} \left(1 + r^f_{t+1} - \frac{q_{it+1}}{q_{it}} \right) \qquad (43a)
$$

and

$$
\frac{Q_{it}}{Q_{it+1}} = \frac{V_{it+1} + \left(1 - \tau^d_{it+1}\right) d_{it+1}}{V_{it}}. \qquad (44a)
$$

The transversality condition $\lim_{T\to\infty} Q_{iT+1} V_{iT+1} = 0$ implies that the price of the stock equals the present value of future dividends,

$$
V_{it} = \sum_{s=0}^{\infty} \left(1 - \tau^d_{it+1+s}\right) Q_{it+1+s} d_{it+1+s}, \qquad (45)
$$

Using the no-Ponzi scheme condition, the budget constraints of the household, (41a) and (42), can be consolidated into a single budget constraint,

$$
\sum_{t=0}^{\infty} Q_{it} \left[q_{it} \left(1 + \tau^c_{it}\right) c_{it} - \left(1 - \tau^n_{it}\right) w_{it} n_{it} \right] \leq \left(1 - l_{i0}\right) a_{i0}, \qquad (46)
$$

[4] Note that we allow only for a tax on wealth in period zero. It turns out that allowing for taxes on wealth in future periods is equivalent to a consumption tax. Since we allow for consumption taxes, taxes on future wealth are redundant.

where

$$a_{i0} = Q_{i-1} b_{i0} + V_{i0} s_{i0} + (1 - \tau^d_{i0}) d_{i0} s_{i0} + \left[1 + r^f_0 - \tau^f_{i0}\left(1 + r^f_0 - \frac{q_{i0}}{q_{i-1}}\right)\right] f_{i0} \tag{47}$$

Substituting for the price of the stock from (45) for $t = 0$, and using (40) as well as $s_0 = 1$, the initial asset holdings in (47) can be written as

$$a_{i0} = Q_{i-1} b_{i0} + \left(1 - \tau^d_{i0}\right) q_{i0} \left[k_0 + \left(1 - \tau^k_{i0}\right)\left(G^i_{i,0} F_{ik,0} - \delta\right) k_{i0}\right] + \left[1 + r^f_0 - \tau^f_{i0}\left(1 + r^f_0 - \frac{q_{i0}}{q_{i-1}}\right)\right] f_{i0}$$

The interest rate parity condition is obtained from (39) and (43a), for $i = 1, 2$,

$$1 + r^f_{t+1} = \frac{q_{it+1}}{q_{it}} \frac{\left(1 - \tau^d_{it+1}\right)}{1 - \tau^d_{it}} \left[\frac{1 - \frac{\left(1 - \tau^d_{it}\right)}{1 - \tau^d_{it+1}} \tau^f_{it+1}}{1 - \tau^f_{it+1}} + \frac{1 - \tau^k_{it+1}}{1 - \tau^f_{it+1}} \left(\frac{p_{it+1}}{q_{it+1}} F^i_{k,t+1} - \delta\right)\right] \text{ for } i = 1, 2. \tag{48}$$

Using also (22) to replace the relative prices of the intermediate and final goods, it follows that

$$\frac{G^1_{j,t}}{G^1_{j,t+1}} \frac{\left(1 - \tau^d_{1t+1}\right)}{1 - \tau^d_{1t}} \left[\frac{1 - \frac{\left(1 - \tau^d_{1t}\right)}{1 - \tau^d_{1t+1}} \tau^f_{1t+1}}{1 - \tau^f_{1t+1}} + \frac{1 - \tau^k_{1t+1}}{1 - \tau^f_{1t+1}} \left(G^1_{1,t+1} F^1_{k,t+1} - \delta\right)\right] \tag{49}$$

$$= \frac{G^2_{j,t}}{G^2_{j,t+1}} \frac{\left(1 - \tau^d_{2t+1}\right)}{1 - \tau^d_{2t}} \left[\frac{1 - \frac{\left(1 - \tau^d_{2t}\right)}{1 - \tau^d_{2t+1}} \tau^f_{2t+1}}{1 - \tau^f_{2t+1}} + \frac{\left(1 - \tau^k_{2t+1}\right)}{1 - \tau^f_{2t+1}} \left(G^2_{2,t+1} F^2_{k,t+1} - \delta\right)\right], \text{ for } j = 1, 2.$$

If the dividend tax is constant over time, and the two tax rates, on capital and foreign assets, are the same in each country, then (dynamic) production efficiency is achieved. Having the tax rates on the two assets, capital and foreign, equal also means that, for the country in which foreign assets are negative, there is a full deductibility of the taxes paid on the capital that is borrowed from abroad.

The marginal conditions in this economy can be summarized as

$$-\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{\left(1 + \tau^c_{it}\right)}{\left(1 - \tau^n_{it}\right) G^i_{i,t} F^i_{n,t}}. \tag{50}$$

$$\frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = \frac{\left(1+\tau^c_{it}\right)}{\left(1+\tau^c_{it+1}\right)} \frac{\left(1-\tau^d_{it+1}\right)}{1-\tau^d_{it}} \left[1 + \left(1-\tau^k_{it+1}\right)\left(G^i_{i,t+1}F^i_{k,t+1} - \delta\right)\right] \tag{51}$$

together with the interest rate parity condition, (49) and condition (8), repeated here,

$$\frac{G^1_{2,t}}{G^1_{1,t}} = \frac{G^2_{2,t}}{G^2_{1,t}}$$

for all $t \geq 0$.

Consider the tax systems that do not tax either consumption or labor, but do have the other taxes. We refer to a tax system in which consumption taxes are set to zero, as a *no-consumption* tax system, and similarly for the labor tax. The proposition follows. The proof is straightforward.

Proposition 5: **(Taxation of assets)** None of the tax systems considered here give higher welfare than the tax system with only consumption and labor income taxes. The Ramsey equilibrium under the no-consumption or the no-labor income tax system requires the taxation of either capital income or dividends and the taxation of foreign assets. With constant dividend taxes, the rates on capital income and foreign assets in each country must be equated. For standard macro preferences, only the consumption or the labor tax is used, and it is constant over time.

3.1.1 Residence based versus source based taxation

In a residence based taxation system, the residents of a country pay taxes to the government of that country on income they may obtain from different sources, from home or abroad. Instead in an source based system they pay taxes in the country that is the source of the income. If there were no further restrictions on the tax rates, whether the system is residence based or source based would not matter. We compute cooperative solutions, allowing for transfers across governments, so what is important is what the tax rates are, not who sets them or gets the revenue from. But a residence based system, in which the residents of a country could be taxed at different rates on their income from home or abroad, is not exactly a residence based system.

We consider now a residence based system in which the tax rates cannot depend on the source of income, whether from home or abroad. In this set up, where firms are domestically owned, this could mean that the tax on capital income would be restricted to be the same as the rate on income from foreign assets. Dividends would not be taxed to avoid double taxation. For the Ramsey allocation, the restriction would not be binding. Instead, in an source based system, if the tax rates have to be the same independently of the residence of the income earner, then it is possible that the tax on capital in one country would have to be the same as the tax on the return of investors in that country. This would be a restriction to the Ramsey problem.

For standard macro preferences, because there is no role for the taxation of assets, the type of system is irrelevant. The proposition follows.

Proposition 6: (Origin vs residence based taxation) Consider the no-consumption and no-labor tax systems with the additional restriction that tax rates have to be the same for all assets. Then, under residence based taxation, the Ramsey equilibrium without that restriction can be implemented, while that is in general not possible under source based taxation. For standard macro preferences, residence or origin based taxation are equivalent.

3.2 Taxes on capital, equity returns and foreign assets

We now consider in addition to capital income taxes, a common tax on the returns from foreign assets and on the equity returns including capital gains. This is a residence based system where capital from different sources is treated the same. We assume that firms are residents of the country where they produce.

We now describe the problems of the firms and the household in each country and define a competitive equilibrium. We maintain the assumption that ownership of firms is domestic, but we will see that this is without loss of generality.

Firm The representative intermediate good firm in each country produces and invests order to maximize the present value of dividends, $V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t d_{it}$. Dividends, in units of the numeraire, d_{it}, are given by

$$d_{it} = p_{it}F(k_{it}, n_{it}) - w_{it}n_{it} - \tau_{it}^k \left[p_{it}F(k_{it}, n_{it}) - w_{it}n_{it} - q_{it}\delta k_{it}\right] - q_{it}\left[k_{it+1} - (1-\delta)k_{it}\right] \tag{52}$$

where τ_{it}^k is the tax rate on capital income net of depreciation.

The first order conditions of the firm's problem are now (20) together with

$$\frac{Q_t q_{it}}{Q_{t+1} q_{it+1}} = 1 + \left(1 - \tau_{it+1}^k\right)\left(\frac{p_{it+1}}{q_{it+1}} F_{k,t+1}^i - \delta\right) \tag{53}$$

Substituting for d_{it} from (38) and using (20) and (53) it is easy to show that the present value of the dividends at time zero in units of the numeraire is given by

$$V_{i0}+d_{i0}=\sum_{t=0}^{\infty}Q_t d_{it}=\left[1+\left(1-\tau_{i0}^{k}\right)\left(\frac{p_{i0}}{q_{i0}}F_{ik,0}-\delta\right)\right]p_{i0}k_{i0}. \tag{54}$$

The problem of the final good firm is as before. The first order conditions are given by (22) .

Households The flow of funds constraint in period t for the household in country i in units of the numeraire is given by

$$\begin{aligned} & b_{it+1}+V_{it}s_{it+1}+f_{it+1} \qquad \text{(55a)}\\ = \; & \frac{Q_{it-1}}{Q_{it}}b_{it}+\left(V_{it}+d_{it}\right)s_{it}-\tau_{it}\left(V_{it}-V_{it-1}+d_{it}-\frac{\left(q_{it}-q_{it-1}\right)V_{it-1}}{q_{it-1}}\right)s_{it}+\\ & \left(1+r_t^f\right)f_{it}-\tau_{it}\left(r_t^f-\frac{q_{it}-q_{it-1}}{q_{it-1}}\right)f_{it}+\left(1-\tau_{it}^{n}\right)w_{it}n_{it}-\left(1+\tau_{it}^{c}\right)q_{it}c_{it}. \end{aligned}$$

In period 0, the constraint is

$$\begin{aligned} & b_{i1}+V_{i0}s_{i1}+f_{i1} \qquad \text{(56)}\\ = \; & \left(1-l_{i0}\right)\left[Q_{i-1}b_{i0}+\left(V_{i0}+d_{i0}\right)s_{i0}-\tau_{i0}\left(V_{i0}-V_{i-1}+d_{i0}-\frac{\left(q_{i0}-q_{i-1}\right)V_{i-1}}{q_{i-1}}\right)s_{i0}\right]+\\ & \left(1-l_{i0}\right)\left[1+r_0^f-\tau_{i0}\left(r_0^f-\frac{q_{i0}-q_{i-1}}{q_{i-1}}\right)\right]f_{i0}+\left(1-\tau_{i0}^{n}\right)w_{i0}n_{i0}-\left(1+\tau_{i0}^{c}\right)q_{i0}c_{i0}. \end{aligned}$$

Dividends and capital gains are taxed at rate τ_{it} with an allowance for numeraire inflation. Returns on foreign assets are also taxed at the same rate, τ_{it}, also with an allowance for numeraire inflation. The returns on public debt, b_{it}, are country specific, $\frac{Q_{it-1}}{Q_{it}}$, because assets can be taxed at different rates in the different countries.

The household's problem is to maximize utility (1) , subject to ($55a$), (56) and no-Ponzi scheme conditions, $\lim_{T\to\infty}Q_{iT+1}b_{iT+1}\geq 0$, and $\lim_{T\to\infty}Q_{iT+1}f_{iT+1}\geq 0$.

The first order conditions of the household's problem in each country are, for $t\geq 0$, (18), and

$$\frac{u_{c,t}^{i}}{\left(1+\tau_{it}^{c}\right)}=\frac{Q_{it}q_{it}}{Q_{it+1}q_{it+1}}\frac{\beta u_{c,t+1}^{i}}{\left(1+\tau_{it+1}^{c}\right)}, \tag{57}$$

together with

$$\frac{Q_{it}}{Q_{it+1}} = (1-\tau_{it+1})\left(1+r^f_{t+1}\right) + \tau_{it+1}\frac{q_{it+1}}{q_{it}} \text{ with } Q_{i0} = 1. \tag{58a}$$

and

$$\frac{Q_{it}}{Q_{it+1}} = \frac{(V_{it+1}+d_{it+1}) - \tau_{it+1}\left(V_{it+1} - V_{it} + d_{it+1} - \frac{q_{it+1}-q_{it}}{q_{it}}V_{it}\right)}{V_{it}} \tag{59a}$$

which implies that

$$1 + r^f_{t+1} = \frac{V_{it+1}+d_{it+1}}{V_{it}} \tag{60a}$$

This condition on the two returns can be written, using $1+r^f_{t+1} = \frac{Q_t}{Q_{t+1}}$, as

$$Q_t V_{it} = Q_{t+1}V_{it+1} + Q_{t+1}d_{it+1}. \tag{61a}$$

Imposing that $\lim_{T\to\infty} Q_{T+1}V_{iT+1} = 0$, then

$$V_{it} = \sum_{s=0}^{\infty}\frac{Q_{t+1+s}}{Q_t}d_{it+1+s}.$$

The present value of dividends for the households of country i, is a different expression from the expression above because they pay taxes on the asset income. Using (59a), we have that

$$V_{i0} = \sum_{t=0}^{\infty}\left(1-\hat{\tau}^a_{it+1}\right)Q_{it+1}d_{it+1}$$

where $1-\hat{\tau}^a_{it+1} = \Pi^t_{s=0}\left(1-\hat{\tau}_{is+1}\right)$, and $1-\hat{\tau}_{it+1} = \frac{(1-\tau_{it+1})}{\left(1-\tau_{it+1}\frac{q_{it+1}Q_{it+1}}{q_{it}Q_{it}}\right)}$. The values are the same since $\left(1-\hat{\tau}^a_{it+1}\right)Q_{it+1} = Q_{t+1}$. This condition is obtained from (58a).

The value of the firm for the households in country i including the dividends in period 0 is

$$\begin{aligned} & V_{i0} + d_{i0} - \tau_{i0}\left(V_{i0} + d_{i0} - \frac{q_{i0}V_{i-1}}{q_{i-1}}\right) \\ = \; & (1-\tau_{i0})(V_{i0}+d_{i0}) + \tau_{i0}\frac{q_{i0}V_{i-1}}{q_{i-1}}. \end{aligned} \tag{63}$$

Notice that the market price of the firm before dividends, $V_{i0}+d_{i0}$, is a linear function of the value for the firm for the households of each country, so that the solution of the maximization problem of the firm also maximizes share holder value. That would also be the case if the stocks were held by the households of the foreign country. This means that the restriction that firms are owned by the domestic households is without loss of generality.

Using the no-Ponzi scheme condition, the budget constraints of the household, (55a) and (56), can be consolidated into the single budget constraint,

$$\sum_{t=0}^{\infty} Q_{it} \left[q_{it} \left(1+\tau_{it}^{c}\right) c_{it} - \left(1-\tau_{it}^{n}\right) w_{it} n_{it} \right] = \left(1-l_{i0}\right) a_{i0}, \tag{64}$$

where

$$a_{i0} = Q_{i-1} b_{i0} + \left(1-\tau_{i0}\right)\left(V_{i0}+d_{i0}\right) + \tau_{i0} \frac{q_{i0} V_{i-1}}{q_{i-1}} + \left[1 + r_0^f - \tau_{i0}\left(1 + r_0^f - \frac{q_{i0}}{q_{i-1}}\right)\right] f_{i0} \tag{65}$$

Using (54) as well as $s_0 = 1$, the initial asset holdings in (47) can be written as

$$\begin{aligned} a_{i0} &= Q_{i-1} b_{i0} + \left(1-\tau_{i0}\right) q_{i0} \left[k_0 + \left(1-\tau_{i0}^{k}\right)\left(G_{i,0}^{i} F_{ik,0} - \delta\right) k_{i0}\right] + \tau_{i0} \frac{q_{i0} V_{i-1}}{q_{i-1}} \\ &\quad + \left[1 + r_0^f - \tau_{i0}\left(1 + r_0^f - \frac{q_{i0}}{q_{i-1}}\right)\right] f_{i0} \end{aligned}$$

The interest rate parity condition is obtained from

$$\frac{Q_t}{Q_{t+1}} = \frac{q_{it+1}}{q_{it}} \left[1 + \left(1-\tau_{it+1}^{k}\right)\left(\frac{p_{it+1}}{q_{it+1}} F_{k,t+1}^{i} - \delta\right)\right] \tag{66}$$

for $i = 1, 2$, or

$$\frac{q_{1t+1}}{q_{1t}} \left[1 + \left(1-\tau_{1t+1}^{k}\right)\left(\frac{p_{1t+1}}{q_{1t+1}} F_{k,t+1}^{1} - \delta\right)\right] = \frac{q_{2t+1}}{q_{2t}} \left[1 + \left(1-\tau_{2t+1}^{k}\right)\left(\frac{p_{2t+1}}{q_{2t+1}} F_{k,t+1}^{2} - \delta\right)\right] \tag{67}$$

Using (22) to replace the relative prices of the intermediate and final goods, it follows that

$$\begin{aligned} &\frac{G_{j,t}^{1}}{G_{j,t+1}^{1}} \left[1 + \left(1-\tau_{1t+1}^{k}\right)\left(G_{1,t+1}^{1} F_{k,t+1}^{1} - \delta\right)\right] \qquad (68) \\ = &\frac{G_{j,t}^{2}}{G_{j,t+1}^{2}} \left[1 + \left(1-\tau_{2t+1}^{k}\right)\left(G_{2,t+1}^{2} F_{k,t+1}^{2} - \delta\right)\right], \text{ for } j = 1, 2 \end{aligned}$$

To get production efficiency, i.e. satisfy (8), need either to set the two tax rates to zero or to pick τ^k_{1t+1} and τ^k_{2t+1} according to

$$\begin{aligned} & \tau^k_{1t+1}\left(G^1_{1,t+1}F^1_{k,t+1}-\delta\right) \\ = \quad & \tau^k_{2t+1}\left(G^1_{1,t+1}F^1_{k,t+1}-\delta-\left(\frac{G^1_{j,t+1}/G^2_{j,t+1}}{G^1_{j,t}/G^2_{j,t}}-1\right)\right),\text{ for } j=1,2 \end{aligned} \tag{69}$$

In order to ensure production efficiency there has to be an adjustment to the movements in the real exchange rate. The tax rate revenue on the return on capital in the consumption of one country must be equal to the tax revenue on the return on capital in the consumption of the other country minus the proportionate change in the real exchange rate.

Using the intertemporal condition of the household (57), and

$$\frac{Q_{it}}{Q_{it+1}}=\left(1-\tau_{it+1}\right)\frac{Q_t}{Q_{t+1}}+\tau_{it+1}\frac{q_{it+1}}{q_{it}} \tag{70a}$$

obtained from (58a) together with $\frac{Q_t}{Q_{t+1}}=1+r^f_{t+1}$, and combining it with the firm's condition (53), together with (22), we obtain

$$\frac{u^i_{c,t}\left(1+\tau^c_{it+1}\right)}{\beta u^i_{c,t+1}\left(1+\tau^c_{it}\right)}=1+\left(1-\tau_{it+1}\right)\left(1-\tau^k_{it+1}\right)\left(G^i_{i,t+1}F^i_{k,t+1}-\delta\right). \tag{71}$$

The marginal conditions in this economy can be summarized by

$$-\frac{u^i_{c,t}}{u^i_{n,t}}=\frac{\left(1+\tau^c_{it}\right)}{\left(1-\tau^n_{it}\right)G^i_{i,t}F^i_{n,t}}. \tag{72}$$

the intertemporal condition (71), the interest rate parity condition, (68) and condition (8), for all $t\geq 0$.

In this economy with a common tax on equity and foreign returns, it is possible to set to zero either the consumption tax or the labor income tax, but not both. The Ramsey allocation can be implemented with a, possibly time varying, common tax on home and foreign assets. Capital income taxes in both countries must either be set to zero, or must be set according to the difference in real returns in the goods of the two countries to ensure production efficiency. For standard macro preferences, all the taxes on assets are set to zero and the labor income tax is constant over time. In this economy with a common tax on domestic equity and foreign returns, firms use a common price to value dividends. The restriction that firms are owned by the domestic residents if relaxed would not change the implementable allocations.

Consider the tax systems that do not tax either consumption or labor, but do have the other taxes. We refer to a tax system in which consumption taxes are set to zero, as a *no-consumption* tax system, and similarly for the labor tax. The proposition follows. The proof is straightforward.

Proposition 7: **(Common tax on domestic equity and foreign returns)** None of the tax systems considered here give higher welfare than the tax system with only consumption and labor income taxes. The Ramsey equilibrium under the no-consumption or the no-labor income tax system requires the taxation of domestic and foreign assets at the same rate. Capital income taxes can be set to zero. For standard macro preferences, only the consumption or the labor tax will be used, and it will be constant over time.

3.3 Border-adjusted value-added taxes and labor income taxes

Consider next an economy in which consumption taxes are replaced by value added taxes levied on firms with border adjustment. What border adjustment means is that firms in a country do not pay VAT taxes on exports, and cannot deduct imports. Taxes on assets are set to zero, but not labor income taxes. The value added taxes are denoted by τ_{it}^{v}. The set up is the same as in the economy with only consumption and labor income taxes except that we distinguish prices in this economy with carets. Because taxes on assets are zero, there is a single intertemporal price of the numeraire.

The intermediate good firm now maximizes

$$\sum_{t=0}^{\infty} \hat{Q}_t \left[(\hat{p}_{i1t} y_{i1t} + \hat{p}_{i2t} y_{i2t}) - \hat{w}_{it} n_{it} - \hat{q}_{it} x_{it} \right] \qquad (73)$$
$$- \sum_{t=0}^{\infty} \hat{Q}_t \tau_{it}^{v} \left[\hat{p}_{iit} y_{iit} - \hat{q}_{it} x_{it} \right]$$

subject to (2) and (4), where $\hat{p}_{ijt}$ is the price of the intermediate good produced in country i and sold in country j.

The final goods firm now maximizes

$$\sum_{t=0}^{\infty} \hat{Q}_t \left[\hat{q}_{it} G^i \left(y_{1it}, y_{2it}\right) - \hat{p}_{1it} y_{1it} - \hat{p}_{2it} y_{2it}\right] - \tag{74}$$
$$\sum_{t=0}^{\infty} \hat{Q}_t \tau_{it}^v \left[\hat{q}_{it} G^i \left(y_{1it}, y_{2it}\right) - \hat{p}_{iit} y_{iit}\right]$$

The household problem is the same as above, except that the consumption taxes are set to zero.

The first order conditions of the household's problem now include

$$-\frac{u_{c,t}^i}{u_{n,t}^i} = \frac{\hat{q}_{it}}{\left(1-\tau_{it}^n\right)\hat{w}_{it}},\ t \geq 0. \tag{75}$$

$$u_{c,t}^i = \frac{\hat{Q}_t \hat{q}_{it}}{\hat{Q}_{t+1} \hat{q}_{it+1}} \beta u_{c,t+1}^i,\ t \geq 0, \tag{76}$$

The first order conditions of the firms' problems for an interior solution are

$$\hat{p}_{iit}\left(1-\tau_{it}^v\right) F_{n,t}^i = \hat{w}_{it} \tag{77}$$

$$\hat{Q}_t \hat{q}_{it}\left(1-\tau_{it}^v\right) = \hat{Q}_{t+1} \hat{p}_{iit+1}\left(1-\tau_{it+1}^v\right) F_{k,t+1}^i + \hat{Q}_{t+1} \hat{q}_{it+1}\left(1-\tau_{it+1}^v\right)(1-\delta)) \tag{78}$$

$$\hat{p}_{iit}\left(1-\tau_{it}^v\right) = \hat{p}_{ijt} \tag{79}$$

$$\hat{q}_{it} G_{i,t}^i = \hat{p}_{iit} \tag{80}$$

$$\hat{q}_{it}\left(1-\tau_{it}^v\right) G_{j,t}^i = \hat{p}_{jit},\ \text{for } j \neq i \tag{81}$$

In order to show equivalence between these two tax systems, consider the following prices with value added taxes. Let

$$\hat{q}_{it}\left(1-\tau_{it}^v\right) = q_{it} \tag{82}$$

$$\hat{p}_{iit}\left(1-\tau_{it}^v\right) = p_{it} \tag{83}$$

$$\hat{p}_{ijt} = p_{it},\ j \neq i,\ \hat{w}_{it} = w_{it},\ \hat{Q}_t = Q_t \tag{84}$$

Replacing the prices with caret in the first order conditions in the economy with value added taxes, we get

$$-\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{q_{it}}{\left(1-\tau^v_{it}\right)\left(1-\tau^n_{it}\right)w_{it}},\ t \geq 0. \tag{85}$$

$$u^i_{c,t} = \frac{Q_t q_{it}}{Q_{t+1} q_{it+1}} \frac{\left(1-\tau^v_{it+1}\right)}{\left(1-\tau^v_{it}\right)} \beta u^i_{c,t+1},\ t \geq 0, \tag{86}$$

$$p_{it} F^i_{n,t} = w_{it} \tag{87}$$

$$p_{it} = p_{it} \tag{88}$$

$$Q_t q_{it} = Q_{t+1} p_{it+1} F^i_{k,t+1} + Q_{t+1} q_{it+1} \left(1-\delta\right)) \tag{89}$$

$$q_{it} G^i_{j,t} = p_{jt} \tag{90}$$

These are the same conditions as in the economy with consumption taxes with

$$1-\tau^v_{it} = \frac{1}{1+\tau^c_{it}} \tag{91}$$

The budget constraints of households in the two cases are (13) and

$$\sum_{t=0}^{\infty} \hat{Q}_t \left[\hat{q}_{it} c_{it} - \left(1-\tau^n_{it}\right) \hat{w}_{it} n_{it}\right] \leq \left(1-l_{i0}\right) a_{i,0}, \tag{92}$$

where

$$a_{i,0} = \hat{q}_{i0}\left(1-\tau^v_{i0}\right)\left[1-\delta+G^i_{i,0}F^i_{k,0}\right]k_{i0} + Q_{i,-1}b_{i0} + \left(1+r^f_{i0}\right)f_{i,0}$$

Using the condition establishing the equivalence between the prices in the two economies, (82) and (84), it follows that the budget constraint in the value added economy (92) becomes (13).

The budget constraint of the governments in the value added economy are given by

$$\begin{aligned} &\sum_{t=0}^{\infty} \hat{Q}_t \left[\tau^v_{it}\left[\hat{p}_{iit} y_{iit} - \hat{q}_{it} x_{it}\right] + \tau^v_{it}\left[\hat{q}_{it} G^i\left(y_{1it}, y_{2it}\right) - \hat{p}_{iit} y_{iit}\right] + \left[\tau^n_{it} \hat{w}_{it} n_{it} - q_{it} g_{it}\right]\right] \\ +l_{i0} a_{i0} \quad = \quad & Q_{i,-1} b_{i0} - T_{i0}. \end{aligned} \tag{93}$$

The balance of payments conditions are

$$\sum_{t=0}^{\infty} \hat{Q}_t \left[\hat{p}_{ijt} y_{ijt} - \hat{p}_{jit} y_{jit}\right] = -\left(1 + r_{i0}^f\right) f_{i,0} - T_{i0}. \tag{94}$$

where $\left(1 + r_{10}^f\right) f_{1,0} + \left(1 + r_{20}^f\right) f_{2,0} = 0$.

Since $\hat{p}_{ijt} = p_{it}$, for $j \neq i$, the balance of payments condition coincides with the one with consumption and labor income taxes.

The two economies are equivalent. This is stated in the following proposition.

Proposition 8: **(Value added taxes with border adjustment)** Competitive equilibrium allocations in the economies with consumption and value added taxes coincide if the taxes in the two systems satisfy (91)

3.4 Value-added taxes without border-adjustment: The role of tariffs

Consider next an economy just like the one in the previous section except that value added taxes are levied on firms without border adjustment. This means that the taxation of intermediate goods will be source based. We will also consider tariffs.

The tariff levied by country j on the good imported from the other country i is denoted by τ_{ijt}^y. The value added taxes in country i are denoted by τ_{it}^v. The intermediate goods firm now maximizes

$$\sum_{t=0}^{\infty} \hat{Q}_t \left[\left(1 - \tau_{it}^v\right)\left(\hat{p}_{i1t} y_{i1t} + \hat{p}_{i2t} y_{i2t} - \hat{q}_{it} x_{it}\right) - \hat{w}_{it} n_{it}\right] \tag{95}$$

subject to (2) and (4), where $\hat{p}_{ijt}$ is the price of the intermediate good produced in country i and sold in country j.

The final goods firm in country 1 now maximizes

$$\sum_{t=0}^{\infty} \hat{Q}_t \left(1 - \tau_{1t}^v\right)\left[\hat{q}_{1t} G^1\left(y_{11t}, y_{21t}\right) - \hat{p}_{11t} y_{11t} - \left(1 + \tau_{21t}^y\right) \hat{p}_{21t} y_{21t}\right] \tag{96}$$

and similarly for country 2.

The household problem is the same as above, except that the consumption taxes are set to zero.

The first order conditions of the household's problem are

$$-\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{\hat{q}_{it}}{\left(1-\tau^n_{it}\right)\left(1-\tau^v_{it}\right)\hat{p}_{iit}F^i_{n,t}},\ t \geq 0. \tag{97}$$

$$u^i_{c,t} = \frac{\hat{Q}_t \hat{q}_{it}}{\hat{Q}_{t+1}\hat{q}_{it+1}}\beta u^i_{c,t+1},\ t \geq 0. \tag{98}$$

The first order conditions of the firms' problems for an interior solution are

$$\hat{p}_{iit}\left(1-\tau^v_{it}\right)F^i_{n,t} = \hat{w}_{it} \tag{99}$$

$$\hat{Q}_t\hat{q}_{it}\left(1-\tau^v_{it}\right) = \hat{Q}_{t+1}\hat{p}_{iit+1}\left(1-\tau^v_{it+1}\right)F^i_{k,t+1} + \hat{Q}_{t+1}\hat{q}_{it+1}\left(1-\tau^v_{it+1}\right)(1-\delta)) \tag{100}$$

$$\hat{p}_{iit} = \hat{p}_{ijt} \equiv \hat{p}_{it} \tag{101}$$

$$\hat{q}_{it}G^i_{i,t} = \hat{p}_{iit},\ i = 1,2 \tag{102}$$

$$\hat{q}_{it}G^i_{j,t} = \left(1+\tau^y_{jit}\right)\hat{p}_{jit},\ \text{for } i \neq j \tag{103}$$

We can rearrange the first order conditions as

$$-\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{1}{\left(1-\tau^n_{it}\right)\left(1-\tau^v_{it}\right)G^i_{i,t}F^i_{n,t}},\ t \geq 0. \tag{104}$$

$$u^i_{c,t}\left(1-\tau^v_{it}\right) = \left(1-\tau^v_{it+1}\right)\beta u^i_{c,t+1}\left[G^i_{i,t+1}F^i_{k,t+1} + 1 - \delta)\right]$$

Using (102) and (103), we get

$$\frac{G^1_{2,t}}{G^1_{1,t}} = \frac{\left(1+\tau^y_{21t}\right)G^2_{2,t}}{\left(1+\tau^y_{12t}\right)G^2_{1,t}}. \tag{105}$$

Using (100) and (102), we have that

$$\frac{1-\tau^v_{1t+1}}{1-\tau^v_{1t}}\frac{\hat{q}_{1t+1}}{\hat{q}_{1t}}\left[G^1_{1,t+1}F^1_{k,t+1} + 1 - \delta\right] = \frac{1-\tau^v_{2t+1}}{1-\tau^v_{2t}}\frac{\hat{q}_{2t+1}}{\hat{q}_{2t}}\left[G^2_{2,t+1}F^2_{k,t+1} + 1 - \delta\right]. \tag{106}$$

From (102) and (103) ,the real exchange rate is

$$\frac{\hat{q}_{1t}}{\hat{q}_{2t}} = \frac{\left(1+\tau^y_{21t}\right)G^2_{2,t}}{G^1_{2,t}} = \frac{G^2_{1,t}}{\left(1+\tau^y_{12t}\right)G^1_{1,t}}, \tag{107}$$

so that it follows that

$$\frac{\left(1-\tau^v_{1t+1}\right)\left(1-\tau^v_{2t}\right)\left(1+\tau^y_{21t+1}\right)}{\left(1-\tau^v_{2t+1}\right)\left(1-\tau^v_{1t}\right)\left(1+\tau^y_{21t}\right)}\frac{G^2_{2,t+1}}{G^1_{2,t+1}}\left[G^1_{1,t+1}F^1_{k,t+1}+1-\delta\right]=\frac{G^2_{2,t}}{G^1_{2,t}}\left[G^2_{2,t+1}F^2_{k,t+1}+1-\delta\right]. \tag{108}$$

The marginal conditions are summarized by

$$-\frac{u^i_{c,t}}{u^i_{n,t}}=\frac{1}{\left(1-\tau^n_{it}\right)\left(1-\tau^v_{it}\right)G^i_{i,t}F^i_{n,t}} \tag{109}$$

$$u^i_{c,t}\left(1-\tau^v_{it}\right)=\left(1-\tau^v_{it+1}\right)\beta u^i_{c,t+1}\left[G^i_{i,t+1}F^i_{k,t+1}+1-\delta)\right] \tag{110}$$

$$\frac{\left(1-\tau^v_{1t+1}\right)\left(1-\tau^v_{2t}\right)\left(1+\tau^y_{21t+1}\right)}{\left(1-\tau^v_{2t+1}\right)\left(1-\tau^v_{1t}\right)\left(1+\tau^y_{21t}\right)}\frac{G^2_{2,t+1}}{G^1_{2,t+1}}\left[G^1_{1,t+1}F^1_{k,t+1}+1-\delta\right]=\frac{G^2_{2,t}}{G^1_{2,t}}\left[G^2_{2,t+1}F^2_{k,t+1}+1-\delta\right] \tag{111}$$

$$\frac{G^1_{2,t}}{G^1_{1,t}}=\frac{\left(1+\tau^y_{21t}\right)G^2_{2,t}}{\left(1+\tau^y_{12t}\right)G^2_{1,t}} \tag{112}$$

The Ramsey allocation in the economy with consumption taxes can be implemented in this economy with a VAT without border adjustment and tariffs. The tariffs will have to be the same in the two countries, $\tau^y_{21t}=\tau^y_{12t}$, to satisfy the (static) production efficiency condition (**??**). The common tariff will in general be time-varying to undo the distortions imposed by the VAT taxes on the (dynamic) production efficiency condition, (7). The value added taxes will have to move over time, differently in the two countries to implement the optimal intertemporal distortions, and the labor income tax, will implement the optimal intratemporal distortion. Without tariffs, the Ramsey allocation in the economy with both consumption and labor income taxes cannot in general be achieved.

For standard macro preferences, there is no need for tariffs, and the Ramsey allocation can be achieved with VAT taxes that, in general, are different across countries, but constant over time. Border tax adjustments in this case are irrelevant.

We state these results in the following proposition:

Proposition 9: **(Value added taxes without border adjustment)** The Ramsey allocation can be implemented with consumption taxes replaced by value added taxes without border adjustment and tariffs. The tariffs must be the same for the two countries and have to be time varying to compensate value added taxes that may move differently across time in the two countries. For standard macro preferences, the value added tax rates are constant over time, and therefore there is no need for tariffs.

Origin versus destination based taxation In order to discuss restrictions on tax systems based on origin and destination, we need to be clear about what a destination based system and an origin based system mean. One possible meaning is the following: A destination based system is one in which taxes are set by the destination country, and similarly, an origin based system is one in which taxes are set by the country where the goods originate from. In such a destination based system there is no reason why imports would be taxed at the same rate as domestically produced goods. Similarly, in an origin based system, there is no reason why exports would be taxed at the same rate as domestically used goods. In such a system, whether destination based, or origin based, there would be four tax rates that would allow to implement the Ramsey allocation. Under the destination based system, the Ramsey policy would set rate on imports equal to the rate on domestically produced goods, and under the origin based system, the rate on exports would be equal to the rate on the goods produced in the destination country.

Another interpretation of destination versus source based systems, that is more restrictive, but it is also closer to what people have in mind, is that a destination based system is one where tax rates do not depend on origin and an origin based system is one where tax rates do not depend on destination. In this case the VAT system with border adjustment, would be a destination based system while the VAT system without border adjustment would be an origin based system. In the case of value added taxes with border adjustment, the goods leave the country untaxed and are taxed in the destination country at the single value added tax rate in the destination country. Instead in the case with value added taxes without border adjustments, goods are taxed at the single rate of the origin country. For this interpretation of destination and source based systems, while the destination based system does not impose relevant restrictions on the set of implementable allocations, the source based system, would in general impose such restrictions. Without tariffs, the destination based system is superior, since in general it is not possible to implement the Ramsey allocation without tariffs when no border adjustments are made. Those restrictions would be undone by tariffs but what tariffs would do is convert an origin based system into a destination based one.

4 On the confiscation of the initial capital stock

The assumption that initial wealth was given in units of utility was useful to allow us to abstract from the initial confiscation of the household's wealth. This is an important assumption because as we will see below, it is the initial confiscation of wealth that justifies high taxation of capital along the transition and possibly asymptotically that have been obtained in the literature as in ...

In this section we discuss alternative assumptions on the initial confiscation and relate our results to this literature. In order to simplify the analysis, and to make it more comparable to the literature, we consider the model with only one country. The production structure is also simplified since, without trade, there is no need to distinguish between an intermediate and a final good. In the economy with two countries we found it convenient to consider a common nominal numeraire. That is no longer the case in this single country economy, so we pick the good in period zero as the numeraire. We consider the standard neoclassical growth model in the closed economy. The remainder of the paper will use this structure

The preferences of a representative household are over consumption c_t and labor n_t, as in (1) without the country indices.

The production technology is

$$c_t + g_t + k_{t+1} - (1-\delta)\, k_t \leq F\left(n_t, k_t\right). \tag{113}$$

We allow for a *rich tax system* similar to the one in the economy in section 3.1 that includes taxes on consumption τ_t^c, labour income τ_t^n, capital income τ_t^k, dividends τ_t^d, and a tax on initial wealth, l_0..

Capital accumulation is conducted by a representative firm. We now describe the problems of the firm and the household and define a competitive equilibrium.

Firm The representative firm produces and invests in order to maximize the present value of dividends, net of taxes, $\sum_{t=0}^{\infty} q_t \left(1-\tau_t^d\right) d_t$, where q_t is the price of one unit of the good produced in period t in units of the good in period zero and τ_t^d are dividend taxes. Dividends, d_t, are given by

$$d_t = F(k_t, n_t) - w_t n_t - \tau_t^k \left[F(k_t, n_t) - w_t n_t - \delta k_t\right] - \left[k_{t+1} - (1-\delta) k_t\right] \tag{114}$$

where w_t is the pre-tax wage rate, τ_t^k is the tax rate on capital income net of depreciation.

Let the interest rate between periods t and $t+1$ be defined by

$$\frac{q_t}{q_{t+1}} \equiv 1 + r_{t+1}, \text{ with } q_0 = 1. \tag{115}$$

The first order conditions of the firm's problem are

$$F_{n,t} = w_t \tag{116}$$

$$1 + r_{t+1} = \frac{\left(1 - \tau_{t+1}^d\right)\left[1 + \left(1 - \tau_{t+1}^k\right)\left(F_{k,t+1} - \delta\right)\right]}{1 - \tau_t^d} \tag{117}$$

where $F_{n,t}$ and $F_{k,t}$ denote the marginal products of capital and labor in period t.

Substituting for d_t from (114) and using (115) – (117) it is easy to show that the present discounted value of dividends is given by

$$\sum_{t=0}^{\infty} q_t \left(1 - \tau_t^d\right) d_t = \left(1 - \tau_0^d\right)\left[1 + \left(1 - \tau_0^k\right)\left(F_{k,0} - \delta\right)\right] k_0. \tag{118}$$

Household The flow of funds constraint in period t for the household is given by

$$\frac{1}{1 + r_{t+1}} b_{t+1} + v_t s_{t+1} = b_t + v_t s_t + (1 - \tau_t^d) d_t s_t + \left(1 - \tau_t^n\right) w_t n_t - \left(1 + \tau_t^c\right) c_t \tag{119a}$$

for $t \geq 1$, and for period 0

$$\begin{aligned} \frac{1}{1 + r_1} b_1 + v_0 s_1 \quad &= \quad \left(1 - l_0\right)\left[b_0 + v_0 s_0 + (1 - \tau_0^d) d_0 s_0\right] + \\ & \quad\quad \left(1 - \tau_0^n\right) w_0 n_0 - \left(1 + \tau_0^c\right) c_0. \end{aligned} \tag{120}$$

where b_{t+1} denotes holdings of government debt that pay one unit of consumption in period $t+1$, s_{t+1} denotes the household's holdings of the shares of the firm, v_t is the price per unit of firm's shares in units of the good in period t.[5] Note that the price v_t is the price of shares after dividends have been paid in period t. The initial conditions

[5] Note that we allow only for a tax on wealth in period zero. It turns out that allowing for taxes on wealth in future periods is equivalent to a consumption tax. Since we allow for consumption taxes, taxes on future wealth are redundant.

are given by b_0 and $s_0 = 1$.

The first order conditions of the household's problem include

$$-\frac{u_{c,t}}{u_{n,t}} = \frac{(1+\tau_t^c)}{(1-\tau_t^n)\,w_t},\ t \geq 0. \tag{121}$$

$$\frac{u_{c,t}}{(1+\tau_t^c)} = (1+r_{t+1})\,\frac{\beta u_{c,t+1}}{\left(1+\tau_{t+1}^c\right)},\ t \geq 0, \tag{122}$$

and

$$1 + r_{t+1} = \frac{v_{t+1} + \left(1-\tau_{t+1}^d\right) d_{t+1}}{v_t}, \tag{123}$$

for all t, where $u_{c,t}$ and $u_{n,t}$ denote the marginal utilities of consumption and labor in period t.

The transversality condition implies that the price of the stock equals the present value of future dividends,

$$v_t = \sum_{s=0}^{\infty} \frac{q_{t+1+s}}{q_t} \left(1-\tau_{t+1+s}^d\right) d_{t+1+s}, \tag{124}$$

Using the no-Ponzi scheme condition, the budget constraints of the household, (119a) and (120), can be consolidated into a single budget constraint,

$$\sum_{t=0}^{\infty} q_t \left[(1+\tau_t^c)\,c_t - (1-\tau_t^n)\,w_t n_t\right] \leq (1-l_0)\left[b_0 + v_0 s_0 + (1-\tau_0^d) d_0 s_0\right]. \tag{125}$$

Substituting for the price of the stock from (124) for $t = 0$, and using (118) as well as $s_0 = 1$, the budget constraint can be written as

$$\sum_{t=0}^{\infty} q_t \left[(1+\tau_t^c)\,c_t - (1-\tau_t^n)\,w_t n_t\right] \leq W_0 \tag{126}$$

where the initial wealth of the household, excluding the lump sum transfer, is given by

$$W_0 \equiv (1-l_0)\left[b_0 + \left(1-\tau_0^d\right)\left[k_0 + \left(1-\tau_0^k\right)(F_{k,0} - \delta)\,k_0\right]\right].$$

A *competitive equilibrium* for this economy consists of a set of allocations $\{c_t, n_t, d_t\}$ and $\{k_{t+1}, b_{t+1}, s_{t+1}\}$, prices $\{q_t, r_{t+1}, v_t, w_t\}$, and policies $\left\{\tau_t^c, \tau_t^n, \tau_t^d, \tau_t^k, l_0\right\}$, given $\{k_0, b_0, s_0\}$

such that households maximize utility subject to their constraints, firms maximize value and markets clear in that resource constraints (113) are satisfied and the market for shares clears, $s_t = 1$ for all t.

The first order conditions associated with the equilibrium when lump-sum taxes are available are given by

$$-\frac{u_{c,t}}{u_{n,t}} = \frac{1}{F_{n,t}}, \tag{127}$$

$$\frac{u_{c,t}}{\beta u_{c,t+1}} = 1 + F_{k,t+1} - \delta, \tag{128}$$

$$\frac{u_{n,t}}{\beta u_{n,t+1}} = \frac{F_{n,t}}{F_{n,t+1}} \left[1 - \delta + F_{k,t+1}\right], \tag{129}$$

and the resource constraints (113). As before, we write the intertemporal labor margin in (129) because we will be interested in understanding when it is optimal not to distort this margin.

With distorting taxes, we can combine the first order conditions of the household and the firm to obtain

$$-\frac{u_{c,t}}{u_{n,t}} = \frac{(1+\tau_t^c)}{(1-\tau_t^n)\, F_{n,t}}, \tag{130}$$

$$\frac{u_{c,t}}{\beta u_{c,t+1}} = \frac{(1-\tau_{t+1}^d)(1+\tau_t^c)}{\left(1-\tau_t^d\right)\left(1+\tau_{t+1}^c\right)} \left[1 + \left(1-\tau_{t+1}^k\right)\left[F_{k,t+1} - \delta\right]\right], \tag{131}$$

and

$$\frac{u_{n,t}}{\beta u_{n,t+1}} = \frac{(1-\tau_{t+1}^d)(1-\tau_t^n)}{\left(1-\tau_t^d\right)\left(1-\tau_{t+1}^n\right)} \frac{F_{n,t}}{F_{n,t+1}} \left[1 + \left(1-\tau_{t+1}^k\right)\left[F_{k,t+1} - \delta\right]\right] \tag{132}$$

Notice that a constant dividend tax does not distort any of the marginal conditions. Such a tax of course raises revenues by reducing the value of the firm at the beginning of period zero. In this sense, a constant dividend tax is equivalent to a levy on the initial capital stock. Notice also that a tax on capital income distorts intertemporal decisions in the same way as do time varying taxes on consumption, dividends and labor income. Indeed, as shown below, many tax systems can implement the same allocations.

Implementability In order to characterize the Ramsey equilibrium we begin by characterizing the set of implementable allocations. In order to do so, we substitute prices and taxes from the first order conditions for the household into the household's budget constraint (126) to obtain

$$\sum_{t=0}^{\infty} \beta^t \left[u_{c,t}c_t + u_{n,t}n_t\right] = \mathcal{W}_0 \tag{133}$$

where

$$\mathcal{W}_0 = \frac{u_{c,0}}{(1+\tau_0^c)} W_0. \tag{134}$$

Thus, any implementable allocations together with initial conditions and period zero policies must satisfy (133) and the resource constraints (113). We now show that the converse also holds. Specifically, consider an arbitrary allocation, that together with initial conditions and period zero policies, satisfies (133) and (113). We will show that this allocation is implementable. To do so, we construct the remaining elements of the allocation, prices and policies and show that all the conditions of a competitive equilibrium are satisfied.

Since multiple tax systems can implement the same allocation, for simplicity we begin by considering the case where $\tau_t^k = 0$ for $t \geq 1$ and $\tau_t^c = \tau_0^c$ for all t. The wage rates w_t are pinned down by (116), and the tax rate on labor τ_t^n is pinned down by (130). Given τ_0^d, the time path of dividend taxes is pinned down by (131) while the time path of consumption prices q_t for $t \geq 1$ is determined by (115) and (117), given $q_0 = 1$. Finally, (124) determines the stock prices p_t, the household's flow of funds determines debt holdings b_{t+1} and dividends d_t are given by (114). It is immediate that these allocations satisfy all the marginal conditions for households and firms. The lump sum transfers are chosen to satisfy the household budget constraint. Thus, the so constructed allocation, prices and policies are a competitive equilibrium.

We summarize this discussion in the following proposition

Proposition 10: (Implementable allocations). Any implementable allocation satisfies the implementability constraint (133) and the resource constraints (113). Furthermore, if a sequence $\{c_t, n_t, k_{t+1}\}$, initial conditions k_0, b_0 and period zero policies $(\tau_0^c, \tau_0^d, \tau_0^k, l_0)$,satisfies (133) and (113), it is implementable.

We emphasize that each implementable allocation can be implemented in numerous ways. For example, consider a tax system which arbitrarily specifies a sequence of taxes on capital income $\widetilde{\tau}_t^k$. The other taxes can be constructed using a similar procedure to the one described above. Alternatively, if taxes on dividends are set to zero, time varying taxes on consumption and labor can be chosen to implement the same allocations.

Given that capital taxes here are redundant instruments, what does it mean that capital should not be taxed? In our view, the relevant question is whether it is optimal to have no intertemporal distortions.

Next we consider restrictions on tax rates. One common practice is to impose an upper bound on the capital income tax. One justification for this upper bound is that the tax revenue ought not to exceed the base, so that $\tau_t^k \leq 1$. Such restrictions are imposed in Chamley (1986) and Judd (1985), Bassetto and Benhabib (2006) or Straub and Werning (2015). These restrictions do not affect the set of implementable allocations because with a rich tax system there are alternative taxes.

Note that analogous restrictions on labor and dividend taxes such as $\tau_t^n \leq 1$, $\tau_t^d \leq 1$ do not restrict the set of implementable allocations. This result follows immediately from inspecting (130) and (131).

4.1 Ramsey equilibrium

We start by assuming that policies and initial conditions are restricted in the sense that households must be allowed to keep an exogenous value of initial wealth $\bar{\mathcal{W}}$, measured in units of utility.

The first order necessary conditions for an interior solution to the Ramsey problem are

$$-\frac{u_{c,t}}{u_{n,t}} = \frac{1+\varphi\left[1+\sigma_t^n-\sigma_t^{nc}\right]}{1+\varphi\left[1-\sigma_t-\sigma_t^{cn}\right]}\frac{1}{F_{nt}},\ t\geq 0 \tag{135}$$

$$\frac{u_{c,t}}{\beta u_{c,t+1}} = \frac{1+\varphi\left[1-\sigma_{t+1}-\sigma_{t+1}^{cn}\right]}{1+\varphi\left[1-\sigma_t-\sigma_t^{cn}\right]}\left[1+F_{k,t+1}-\delta\right],\ t\geq 0 \tag{136}$$

$$\frac{u_{n,t}}{\beta u_{n,t+1}} = \frac{1+\varphi\left(1+\sigma_{t+1}^n-\sigma_{t+1}^{nc}\right)}{1+\varphi\left(1+\sigma_t^n-\sigma_t^{nc}\right)}\frac{F_{n,t}\left[1+F_{k,t+1}-\delta\right]}{F_{n,t+1}},\ t\geq 0 \tag{137}$$

together with the constraints, where φ is the multiplier of the implementability condition.

Comparing these conditions (135) – (137) with the related conditions with lump-sum taxes (127) – (129) it is clear that the optimal wedges depend on own and cross elasticities of consumption and labor. If those elasticities are constant, it is optimal not to have intertemporal distortions. Note that in this case intratemporal wedges are constant and in general positive.

Note that conditions (136) and (137) imply that if elasticities are not constant over time, it is optimal to have intertemporal distortions, but whether it is optimal effectively to tax or subsidize capital accumulation depends on whether elasticities are increasing or decreasing over time.

Note also that if consumption and labor are constant over time, then the relevant elasticities are also constant so that it is optimal to have no intertemporal distortions.

Propositions analogous to propositions 3 and 4 trivially follow: If the Ramsey equilibrium converges to a steady state, it is optimal to have no intertemporal distortions asymptotically. And, for standard macro preferences, the Ramsey solution has no intertemporal distortions for all $t \geq 0$.

Suppose next that policies are restricted in that the household must keep at least $\mathcal{V}_0$ initial wealth, but now in units of goods rather than in utility terms. This *wealth restriction in goods units* implies that the constraint faced by the Ramsey planner on the confiscation of initial wealth is

$$\frac{W_0}{1+\tau_0^c} \geq \bar{\mathcal{V}}.$$

The implementability constraint can then be written as

$$\sum_{t=0}^{\infty} \beta^t \left[u_{c,t}c_t + u_{n,t}n_t\right] \geq u_{c,0}\bar{\mathcal{V}}.$$

The problem is the same as before except for the term on the right hand side. The Ramsey conditions are the same as before in (135), (136) and (137), for all $t \geq 1$. The conditions for period zero are different. The intertemporal condition for consumption between periods zero and one, for example, is now

$$\frac{u_{c,0}}{\beta u_{c,1}} = \frac{1+\varphi\left(1-\sigma_1+\sigma_1^{cn}\right)}{1+\varphi\left(1-\sigma_0+\sigma_0^{cn}+\frac{\sigma_0\bar{\mathcal{V}}}{c_0}\right)}\left[1+F_{k,1}-\delta\right]. \tag{138}$$

and the intratemporal condition at time zero is

$$-\frac{u_{c,0}}{u_{n,0}} = \frac{1+\varphi\left[1+\sigma_0^{n}-\sigma_0^{nc}+\sigma_0^{nc}\frac{\bar{\mathcal{V}}}{c_0}\right]}{1+\varphi\left[1-\sigma_0+\sigma_0^{cn}+\sigma_0\frac{\bar{\mathcal{V}}}{c_0}\right]}\frac{1}{F_{n,0}} \tag{139}$$

Since the Ramsey conditions for $t \geq 1$ are unaffected, as before, wether it is optimal effectively to tax or subsidize capital accumulation depends on wether elasticities are increasing or decreasing over time.

With standard macro preferences, since elasticities are constant over time, it is optimal to have no intertemporal distortions, from period one onwards. Consider now intertemporal distortions in period zero. With standard macro preferences $\sigma_1 = \sigma_0$ and the cross elasticities are zero, so that if $\bar{\mathcal{V}} > 0$, (138) implies that

$$\frac{u_{c,0}}{\beta u_{c,1}} < 1 + F_{k,1} - \delta.$$

Thus, it is optimal to effectively tax capital accumulation in period zero, or subsidize the consumption good in period zero, relative to consumption in future periods. One intuition for this result is as follows. The household is entitled to an exogenous amount of wealth in period zero. The Ramsey planer finds it optimal to reduce the value of this wealth in utility terms. This value can be reduced by decreasing the marginal utility of period zero consumption. This decrease is achieved by inducing households to increase their period zero consumption relative to consumption in all future periods. We summarize this discussion in the following proposition.

Proposition 11: (No intertemporal distortions after one period) Suppose preferences satisfy (36) and the wealth restriction in goods units must be satisfied. Then the Ramsey solution has no intertemporal distortions for all $t \geq 1$. If $\bar{\mathcal{V}} > 0$, it is optimal to effectively tax capital accumulation from period zero to period one.

The Ramsey allocation can be implemented as follows: Set the initial tax rate on wealth to satisfy the wealth restriction; set capital income and consumption taxes to zero in all periods; set the labor income tax to satisfy (130), (135) and (139). Specifically, set the labor income tax to

$$1 - \tau_0^n = \frac{1 + \varphi \left[1 - \sigma + \frac{\sigma \bar{\mathcal{V}}}{c_0}\right]}{1 + \varphi \left[1 + \sigma^n\right]},$$

in period zero and to

$$1 - \tau^n = \frac{1 + \varphi \left[1 - \sigma\right]}{1 + \varphi \left[1 + \sigma^n\right]}$$

in all future periods. Set the dividend tax to zero in period zero and then to a constant value thereafter. This constant value τ^d must satisfy (138) and (131) so that its value is given by

$$\frac{1+\varphi\left(1-\sigma\right)}{1+\varphi\left(1-\sigma+\frac{\sigma\bar{\mathcal{V}}}{c_0}\right)}=1-\tau^d.$$

Note that under this implementation the tax rate on dividends is always less than one and is positive if $\bar{\mathcal{V}}$ is positive. An alternative implementation uses the consumption tax rather than the dividend tax. The disadvantage of this implementation is that in order to satisfy (130) and (135), the required tax on labor income might have to be negative, to compensate the effect of the higher consumption tax after period 1. The dividend tax implementation has the advantage that this tax affects only intertemporal decisions, so that the labor income tax can be chosen to satisfy the intratemporal condition. The consumption tax has the disadvantage that it affects inter and intratemporal decisions. The dividend tax has the disadvantage that, as we remarked earlier, the base on which it is levied could be negative, so that the tax would constitute a subsidy to the firm.

The standard implementation in the literature uses capital and labor income taxes and sets consumption and dividend taxes to zero. A disadvantage of this implementation is that the capital income tax may have to be greater than 100% to implement the Ramsey allocation. Given this disadvantage, the literature typically imposes an additional restriction that the tax rates on capital income cannot exceed some upper limit $\bar{\tau}$. This restriction implies the following additional constraint to the Ramsey problem

$$\frac{\frac{u_{c,t}}{\beta u_{c,t+1}}-1}{F_{k,t+1}-\delta}\geq 1-\bar{\tau}.$$

This restriction may bind for a number of periods as in Chamley (1986) or forever as in Straub and Werning (2015). Straub and Werning allow the maximum tax rate to be 100% and show that the optimal solution for particularly high levels of initial debt may have the capital income tax be set at 100% forever. One intuition for the Straub and Werning finding is that by taxing capital income forever, real interest rates are zero forever and that is the way consumption in period zero can be increased the most, reducing the value of the good in the initial period. Given that the initial real rate cannot be below zero, the whole term structure is flattened down to zero.

To see this more clearly, notice that the planner has a strong incentive to make $u_{c,0}$ small so as to reduce the value of initial wealth. We refer to this incentive as the confiscation motive. The planner however must respect the intertemporal conditions, with restricted taxes,

$$\frac{u_{c,t}}{\beta u_{c,t+1}} = 1 + \left(1 - \tau_{t+1}^{k}\right)(F_{k,t+1} - \delta).$$

Given $u_{c,1}$, the confiscation motive provides an incentive to make τ_1^k large to reduce $u_{c,0}$. If the confiscation motive is sufficiently strong, the bound on τ_1^k is met. In this case, the planner has an incentive to make $u_{c,1}$ small to reduce $u_{c,0}$, thereby confiscating initial wealth. Fixing $u_{c,2}$, $u_{c,1}$ in turn can be made small by making τ_2^k large. Again, if the confiscation motive is sufficiently strong, the upper bound will be met. This recursion suggests that the Ramsey solution will have capital taxes be at the upper bound for a length of time, and then zero. If the initial debt is sufficiently large, the confiscation motive is very strong, and the length of time could be infinite as pointed out by Straub and Werning.

With a rich tax system, and with fixed initial policies, the confiscation motive is satisfied in one period by levying a sufficiently high dividend tax in period one, as is apparent from (131). The advantage of the dividend tax is that it effectively allows the tax to apply to a larger base than the capital income tax. This advantage can be seen by inspecting (131) evaluated in period zero with zero consumption taxes and zero dividend taxes in period zero. Notice that if the dividend tax in period one was used fully at 100% the gross return on capital would be zero. In contrast, full taxation of capital income with $\tau_1^k = 1$, can only reduce the net return to zero, provided $F_{k,1} - \delta \geq 0$.

4.2 Partial commitment equilibria

The notion of Ramsey equilibrium is developed in an environment in which in period zero the government commits to an infinite sequence of policies. Here we consider an alternative institutional framework in which the government has partial commitment. We develop a notion of equilibrium for such an environment, referred to as a *Partial commitment equilibrium.* In our environment, in any period, governments lack full commitment in the sense that they cannot specify the entire sequence of policies that will be chosen in the future. They do have the ability to constrain the set of policies in the subsequent period. We consider two kinds of constraints.

In the first kind, the government in any period can commit to the one period returns on assets in utility terms. The government in the following period is free to choose policies as it wishes but must respect the previously committed return constraints. In the second kind, the government in any period can commit to (a subset of) policies in the following period. We show that with the first kind of partial commitment the equilibrium coincides with that under full commitment with constraints on the initial value of wealth. With the second kind of partial commitment, equilibrium outcomes do not coincide with those under full commitment with constraints on initial policies.

Consider the environment with partial commitment on returns. In order to develop our notion of partial commitment in this environment consider the intertemporal Euler equations for bonds and capital from period $t-1$ to period t.

$$\frac{u_{c,t-1}}{\beta\left(1+r_{t-1}\right)\left(1+\tau_{t-1}^{c}\right)}=\frac{u_{c,t}}{\left(1+\tau_{t}^{c}\right)}, \tag{140}$$

$$\frac{u_{c,t-1}\left(1-\tau_{t-1}^{d}\right)}{\beta\left(1+\tau_{t-1}^{c}\right)}=\frac{u_{c,t}\left(1-\tau_{t}^{d}\right)\left[1+\left(1-\tau_{t}^{k}\right)\left(F_{k,t}-\delta\right)\right]}{\left(1+\tau_{t}^{c}\right)}. \tag{141}$$

Let $\lambda_{1,t}$ denote the right side of (140), and $\lambda_{2,t}$ denote the right side of (141). With partial commitment, the government in period $t-1$ chooses period $t-1$ policies as well as $\lambda_{1,t}$ and $\lambda_{2,t}$. The government in period t can choose any policies but they must have the property that the induced allocations and policies must satisfy the constraints on returns

$$\lambda_{1,t}=\frac{u_{c,t}}{\left(1+\tau_{t}^{c}\right)} \tag{142}$$

and

$$\lambda_{2,t}=\frac{u_{c,t}\left(1-\tau_{t}^{d}\right)\left[1+\left(1-\tau_{t}^{k}\right)\left(F_{k,t}-\delta\right)\right]}{\left(1+\tau_{t}^{c}\right)}. \tag{143}$$

The government in period t chooses period t policies as well as $\lambda_{1,t+1}$ and $\lambda_{2,t+1}$, to constrain policies in period $t+1$.[6] We assume $\lambda_{1,0}$ and $\lambda_{2,0}$ are given.

In order to understand the nature of partial commitment here, note that the Euler equations for bonds and capital, (140) and (141), will, of course, be satisfied on the equilibrium path. The spirit of this form of partial commitment is that the government must respect these intertemporal Euler equations also off the equilibrium path. This requirement is intended to capture the idea that policies must not induce regret on the part of households on their past choices. Specifically, the government in period t is

[6] We consider the same tax instruments, except that now, in order to treat every period alike, we set the initial wealth tax to zero, $l_0=0$.

free to choose policies as it wishes but since any policy must respect the intertemporal Euler equations it will not induce regret about choices made in period $t-1$. The spirit of the assumption that $\lambda_{1,0}$ and $\lambda_{2,0}$ are given is that the economy was operating in previous periods, and the choices made in those previous periods constrain the choices in period 0 as well.

Next, we develop a notion of Markov equilibrium with partial commitment on returns which we call *non confiscatory equilibrium*. It is convenient, and without loss of generality, to think of the government in period t, as choosing allocation, policies and prices directly in that period. The state of the economy in period t is given by $s_t = \{k_t, b_t, \lambda_{1,t}, \lambda_{2,t}\}$. Let $h_t(s_t)$ denote the policy function in period t which maps the state of the economy into allocation, policies, prices and $\lambda_{1,t+1}$, $\lambda_{2,t+1}$. The government in period t maximizes welfare taking as given the continuation value function and the policy functions in period $t+1$, subject to the marginal conditions of agents, budget constraints and market clearing conditions in period t. Specifically, the government in period t solves the following problem

$$v_t(s_t) = \max\left\{u(c_t, n_t) + \beta v_t(s_{t+1})\right\}, \tag{144}$$

subject to the period t equilibrium conditions and (142) and (143). Note that the policy function $h_{t+1}(s_{t+1})$ enters the intertemporal Euler equations in these equilibrium conditions. For example, period $t+1$ policies on consumption, labor, and the consumption tax, appear in the households bond Euler equation,

$$\frac{u_{c,t}}{(1+\tau_t^c)} = (1+r_t)\frac{\beta u_c\left(c_{t+1}(s_{t+1}), n_{t+1}(s_{t+1})\right)}{\left(1+\tau_{t+1}^c(s_{t+1})\right)} \tag{145}$$

where $c_{t+1}(s_{t+1})$, $n_{t+1}(s_{t+1})$ and $\tau_{t+1}^c(s_{t+1})$ are elements of $h_{t+1}(s_{t+1})$.

A Markov equilibrium with partial commitment on returns, a *non confiscatory equilibrium*, consists of value functions $v_t(s_t)$ and policy functions, $h_t(s_t)$, which solve (144) for all s_t and all t.

Next we show that the Markov equilibrium outcome coincides with the Ramsey outcome with wealth constraints. Using the same logic as in our characterization result in Proposition 1, it is straightforward to show that the period t equilibrium conditions can be equivalently represented by the resource constraint and by the following period

t implementability constraint,

$$\beta\lambda_{1,t+1}b_{t+1} + \beta\lambda_{2,t+1}k_{t+1} = \lambda_{1,t}b_t + \lambda_{2,t}k_t - u_{n,t}n_t - u_{c,t}c_t. \tag{146}$$

Multiplying these constraints by β^t and summing up yields the implementability constraint of the Ramsey problem. Thus the Ramsey allocation is feasible. Note that future controls do not appear in the objective function, (144), or the constraint set which includes (146). We can then use an identical argument to that in Stokey and Lucas with Prescott (1989)[7], to show that the functional equation in (144) solves the date zero sequence problem.

We have proved the following proposition.

Proposition 12: (Partial commitment is full commitment). The Markov outcome of an economy with partial commitment in returns coincides with the Ramsey outcome with wealth restriction given by $W_0 = \lambda_{1,0}b_0 + \lambda_{2,0}k_0$.

Kydland and Prescott (1980) propose a method to compute Ramsey outcomes. They show that a Ramsey equilibrium could be characterized recursively starting in period one, with the addition of a state variable. This state variable represents promised marginal utilities which is the analog to $\lambda_{1,t}$ and $\lambda_{2,t}$ in our environment. The government in period zero maximizes discounted utility while being unconstrained by the added state variable. An extensive literature has exploited this recursive formulation to characterize commitment outcomes. We show here that their clever insight can be used to prove that equilibria in environments where policy makers are constrained not to confiscate coincide with equilibria with full commitment and initial wealth constraints.

A partial commitment equilibrium on instruments Consider next an alternative form of partial commitment. In this form, the government in period t chooses a subset of policies, $\left\{\tau^c_{t+1}, \tau^k_{t+1}, \tau^d_{t+1}\right\}$ that will be implemented in period $t+1$. The government in any period t is free to choose the labor income tax, τ^n_t. The spirit of this assumption is that in the literature, as already discussed, this subset of policies is exogenously fixed in period zero. We extend this spirit to allow for partial commitment to that subset of instruments in every period. The Markov equilibrium, with this form of partial commitment does not in general coincide with the Ramsey outcomes with exogenously specified initial taxes. Together with our results on partial commitment

[7] Theorem 4.3

on returns, this result shows that the nature of partial commitment plays a crucial role in determining whether Markov equilibria coincide with commitment equilibria.

Consider, the implementability constraint with this form of partial commitment. It follows from (142) and (143), that the implementability constraint can be written as (146), above. Notice here that $\lambda_{1,t+1}$ and $\lambda_{2,t+1}$ depend not only on policies chosen in the current period, $\left\{\tau^c_{t+1}, \tau^k_{t+1}, \tau^d_{t+1}\right\}$, but also on allocations and policies that will be chosen in the next period.

A Markov equilibrium is defined analogously to the one above. The state of the economy in period t is given by $s_t = \left\{k_t, b_t, \tau^c_t, \tau^k_t, \tau^d_t\right\}$. As before let $h_t(s_t)$ denote the policy function which maps the state of the economy into allocations, the labor tax rate, prices and the period $t+1$ taxes, $\left\{\tau^c_{t+1}, \tau^k_{t+1}, \tau^d_{t+1}\right\}$. The government in period t solves the analogous problem to the one above.

Note that in a Markov equilibrium, $\lambda_{1,t+1}$, for example, is given by

$$\lambda_{1,t+1} = \frac{u_c\left(c_{t+1}\left(s_{t+1}\right), n_{t+1}\left(s_{t+1}\right)\right)}{\left(1+\tau^c_{t+1}\left(s_t\right)\right)} \tag{147}$$

This equation shows the precise sense in which $\lambda_{1,t+1}$ depends on the policy function which will be followed in in the next period. The government in period t takes this future policy function as given in choosing its current optimal policy. Put differently, future controls appear in the constraint set in period t. The arguments in Stokey and Lucas with Prescott (1989) no longer apply.

Lucas and Stokey (1983) provide examples in production economies without capital where the Ramsey outcome is time inconsistent. Chari and Kehoe (1993) characterize Markov equilibria in that environment. Klein, Krusell, Rios-Rull (2008) characterize Markov equilibria in environments similar to ours, with partial commitment to instruments. The results in these papers imply that Markov outcomes are in general different from commitment outcomes.

5 Heterogeneous agents within a country

The results obtained above for the representative agent economy remain under certain conditions in economies with capital-rich and poor agents. In order to show this consider an economy with equal measure of two types of agents, 1 and 2. The social welfare function is

$$\theta U^1 + (1-\theta) U^2$$

with weight $\theta \in [0,1]$. The individual preferences are assumed to be the standard preferences allowing for possibly different elasticities for the two types of agents,

$$U = \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t^i\right)^{1-\sigma^i} - 1}{1-\sigma} - \eta^i \left(n_t\right)^{\psi^i} \right]. \tag{148}$$

The resource constraints are

$$c_t^1 + c_t^2 + g_t + k_{t+1} - (1-\delta) k_t \leq A_t F\left(n_t^1 + n_t^2, k_t\right),$$

where $k_t = k_t^1 + k_t^2$.

The taxes are the ones in the rich tax system considered in the representative agent economy that includes taxes on consumption τ_t^c, labour income τ_t^n, capital income τ_t^k, dividends τ_t^d and a tax on initial wealth, l_0. Note that we do not allow for the taxes to differ across agents.

With heterogeneous agents it turns out that we do not need to impose constraints on the initial policies. In particular, for reasons pointed out in Werning (2007), it turns out that without constraints on wealth taxes, it may be optimal for the planner to distort intratemporal decisions.

The implementability conditions can be written as

$$\sum_{t=0}^{\infty} \beta^t \left[u_{c,t}^1 c_t^1 + u_{n,t}^1 n_t^1\right] = u_{c,0}^1 (1-l_0) V_0^1, \tag{149}$$

and

$$\sum_{t=0}^{\infty} \beta^t \left[u_{c,t}^2 c_t^2 + u_{n,t}^2 n_t^2\right] = u_{c,0}^2 (1-l_0) V_0^2, \tag{150}$$

with $V_0^i = \left[b_0^i + \left(1-\tau_0^d\right)\left[1 + \left(1-\tau_0^k\right)\left(F_{k,0} - \delta\right)\right] k_0^i\right]$. Since the taxes must be the same for the two agents an implementable allocation must also satisfy the following marginal conditions

$$\frac{u_{c,t}^1}{u_{c,t}^2} = \frac{u_{n,t}^1}{u_{n,t}^2}$$

and

$$\frac{u^1_{c,t}}{u^2_{c,t}} = \frac{u^1_{c,t+1}}{u^2_{c,t+1}}$$

These conditions can be written as

$$u^1_{c,t} = \gamma u^2_{c,t} \tag{151}$$

$$u^1_{n,t} = \gamma u^2_{n,t} \tag{152}$$

where γ is some endogenous number.[8]

Let φ^1 and φ^2 be the multipliers of the two implementability conditions, (149) and (150). The first order conditions for $t \geq 1$ imply[9]

$$u^2_{c,t} \frac{\gamma \left[\theta + \varphi^1 \left(1 - \sigma^1\right)\right] \frac{\sigma^2}{c^2_t} + \left[(1-\theta) + \varphi^2 \left(1 - \sigma^2\right)\right] \frac{\sigma^1}{c^1_t}}{\frac{\sigma^2}{c^2_t} + \frac{\sigma^1}{c^1_t}} = \lambda_t \tag{153}$$

and

$$u^2_{n,t} \frac{\gamma \left[\theta + \varphi^1 \left(1 + \psi^1\right)\right] \frac{\psi^2}{n^2_t} + \left[(1-\theta) + \varphi^2 \left(1 + \psi^2\right)\right] \frac{\psi^1}{n^1_t}}{\frac{\psi^2}{n^2_t} + \frac{\psi^1}{n^1_t}} = -\lambda_t F_{n,t},\ t \geq 1 \tag{154}$$

which, together with

$$-\lambda_t + \beta \lambda_{t+1} \left[f_{k,t+1} + 1 - \delta\right] = 0$$

imply that, if elasticities are equal, $\sigma^1 = \sigma^2 = \sigma$ and $\psi^1 = \psi^2 = \psi$, future capital should not be taxed from period one on. To see this, notice that, from (151) and (152), c^1_t must be proportionate to c^2_t, $c^1_t = \gamma^{-\frac{1}{\sigma}} c^2_t$, and n^1_t must also be proportionate to n^2_t, $n^1_t = (\gamma)^{\frac{1}{\psi}} n^2_t$. It then follows that the terms multiplying the marginal utilities on the left hand side of (153) and (154) are time invariant.

In period zero the first order condition for consumption of type one has an additional term. Using $u^1_{c,0} = \gamma u^2_{c,0}$, that first order condition is

$$\theta u^1_{c,0} + \varphi^1 u^1_{c,0} \left(1 - \sigma^1_0\right) + \mu_0 u^1_{cc,0} - u^1_{cc,0} \left(1 - l_0\right) \left(\varphi^1 V^1_0 + \varphi^2 \frac{V^2_0}{\gamma}\right) = \lambda_0. \tag{155}$$

[8]See also Greulichy, Laczó and Marcet (2016). Werning (2007) also computes optimal taxes with heterogeneous agents taking adavantage of this proportionality of marginal utilities.

[9]See Appendix... for the derivation.

The first order condition for an interior solution of l_0 is

$$-u_c^1(0)\left(\varphi^1 V_0^1 + \varphi^2 \frac{V_0^2}{\gamma}\right) = 0$$

Thus, the additional term is zero, so that the first order condition for period zero, (155), has the same form as the ones for $t \geq 1$.

Consider next the additional restriction that the initial wealth tax has to be lower than 100%, $l_0 \leq 1$. If the solution to this problem is interior, the additional term is zero. If the solution has $l_0 = 1$, the additional term is also zero.

The first order conditions for labor of both types in period zero also have additional terms. The condition for labor of type one in period 0, also using $u_{c,0}^1 = \gamma u_{c,0}^2$, is

$$\theta u_{n,0}^1 + \varphi^1 u_{n,0}^1(1+\psi) + \mu_0^n u_{nn,0}^1 - u_{c,0}^1(1-l_0)\left(1-\tau_0^k\right) F_{kn,0}\left[\varphi^1 k_0^1 - \varphi^2 \frac{k_0^2}{\gamma}\right] = -\lambda_0 F_{n,0} \tag{156}$$

and similarly for n_0^2.

The derivative with respect to τ_0^k is $\varphi^1 u_{c,0}^1(1-l_0)(F_{k,0}-\delta)k_0^1 + \varphi^2 \frac{u_{c,0}^1}{\gamma}(1-l_0)(F_{k,0}-\delta)k_0^2$. If there are no restrictions on τ_0^k, the solution is interior, and then $\varphi^1 k_0^1 + \varphi^2 \frac{k_0^2}{\gamma} = 0$. On the other hand, if τ_0^k is restricted to be below 100%, and if the solution is at the corner, $\tau_0^k = 1$, the term in the first order condition for labor in period zero is again zero.

We summarize this discussion in the following proposition.

Proposition 13: (No intertemporal distortions in heterogenous agent economies) Suppose that preferences for all types of agents are in the class of standard macroeconomic preferences. If all agents have the same preferences, then the Ramsey equilibrium has no intertemporal distortions for all $t \geq 0$.

Note that this proposition holds even if we impose the additional restriction that $l_0 \leq 1$.

This proposition shows that with standard and identical preferences, allowing for heterogeneity in initial wealth does not overturn the result that, with a rich tax system, future capital should not be taxed.

With heterogeneity and distributional concerns it may be optimal for the planner to distort intratemporal decisions regardless of whether the initial wealth tax is constrained to be below 100% or not. This result is in striking contrast with the result in the representative agent model. In that model, as stated in Proposition 2, if the initial wealth tax is unconstrained, the outcome coincides with the lump sum tax allocations and the intratemporal decisions are undistorted.

Consider next the case in which agents' preferences differ, in that $\sigma^1 \neq \sigma^2$. In this case it is possible to show that if the two types of agents can be taxed at different rates, it is optimal not to have intertemporal distortions. The reason is that restrictions (151) and (152) can now be dropped and the analog of proposition 6 can be proved. If the two types of agents cannot be taxed at different rates, then it may be optimal to distort intertemporal decisions.

6 Relating capital taxation to production efficiency

In this section we connect our results to results on production efficiency and uniform taxation. To develop these connections we set up an alternative economy, that we call an *intermediate goods economy* that seems different at face value but turns out to be equivalent to the one considered above. In this alternative economy, the representative household consumes a single final good denoted by C, and supplies a single final labor denoted by N. Preferences for the household are given by

$$U\left(C, N\right) = \frac{C^{1-\sigma} - \frac{1}{1-\beta}}{1-\sigma} - \eta N^{\psi}. \tag{157}$$

The economy has three types of firms. The first one is the same as the one described above. We refer to this as the *capital accumulation firm*. This firm produces intermediate goods c_t, hires intermediate labor inputs n_t, and accumulates capital according to the technology (113). The second type of firm, referred as the *consumption firm* produces the final good C using the intermediate goods c_t according to the constant returns to scale technology given by

$$C = \mathcal{C}(c_0, c_1..) = \left[\sum_{t=0}^{\infty} \beta^t c_t^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{158}$$

The third type of firm, referred to as the *labor firm* produces the intermediate labor inputs using final labor, according to the constant returns to scale technology given by

$$N = \mathcal{N}(n_0, n_1..) = \left[\sum_{t=0}^{\infty} \beta^t n_t^{\psi}\right]^{\frac{1}{\psi}}. \tag{159}$$

In terms of the tax system, we assume that the government can levy a tax on the final consumption good, τ^c, and on final labor denoted by τ^n. In addition, we assume that the government can levy taxes on all intermediate goods, denoted by τ_t^c and τ_t^n. We retain the dividend taxes and the capital income taxes levied on the capital accumulation firm, as well as the initial levy l_0. We do not impose any taxes on the profits of either the consumption firm nor the labor firm because these profits will be zero in equilibrium.

The household's problem is to maximize (157) subject to the budget constraint

$$p(1+\tau^C)C - w(1-\tau^N)N \leq (1-l_0)\,V_0 \tag{160}$$

where p and w denote the prices of final consumption and labor in units of the consumption good in period 0, and V_0 is the value of initial wealth in units of goods.

The consumption firm's problem is to maximize

$$pC - \sum_{t=0}^{\infty} q_t(1+\tau_t^c)c_t \tag{161}$$

subject to (158) .

The labor firm's problem is to maximize

$$\sum_{t=0}^{\infty} q_t(1-\tau_t^n)w_t n_t - wN \tag{162}$$

subject to (159) .

The capital accumulation firm's problem is the same as above. A competitive equilibrium is defined in the standard fashion.

Next, we show that the competitive equilibria in the two economies coincide, in the intermediate goods economy and in the growth economy with distorting taxes. To do so, we first show that we can rewrite the equilibrium in the intermediate goods economy as an equilibrium in the growth economy with taxes by incorporating the decisions of the consumption and the labor firm directly into the households problem.

Consider the budget constraint of the household. Given that in any competitive equilibrium profits are zero for the consumption and the labor firm, we can substitute $pC = \sum_{t=0}^{\infty} q_t(1+\tau_t^c)c_t$ from (161) and $wN = \sum_{t=0}^{\infty} q_t(1-\tau_t^n)w_t n_t$ from (162) into (160) to obtain a budget constraint of the form (126) in the growth economy with taxes. The only difference is the presence of the tax on the final consumption and on final labor, which amounts to rescaling the consumption and labor income taxes in the original economy.

Substituting from (159) and (158) into (157), we see that the household's utility function in the rewritten intermediate goods economy is the same as in the growth economy with taxes.

To establish that the converse holds, note that we can set up the household's problem in the growth economy with taxes as a two stage problem of first choosing an aggregate value for consumption and labor and then choosing the disaggregated levels of consumption and labor to achieve the desired value of the final consumption and labor. Thus, the equilibria in the two economies coincide.

In the intermediate goods economy, the Ramsey problem is to maximize (157) subject to the implementability constraint

$$U_C C + U_N N = \frac{U_C}{(1+\tau^C)} (1-l_0) V_0, \tag{163}$$

and the requirement that the allocation is in the *production set* given by (158) and (159) with inequalities and (113) .

Suppose now, as before, that the initial wealth in utility terms, given by the right hand side of (163) must be at least as large as an exogenous value, $\bar{\mathcal{W}}$.

Next we apply the production efficiency theorem of Diamond and Mirrlees to our intermediate goods economy. This theorem asserts that if pure rents are fully taxed away, any Ramsey allocation must be at the boundary of the production set. In our context, the theorem asserts that if $\bar{\mathcal{W}} = 0$, any Ramsey allocation must be at the boundary of the production set. In the Appendix we extend this theorem to the case where $\bar{\mathcal{W}}$ is exogenously fixed.

Somewhat loosely speaking, the theorem as extended, implies that one way to implement the Ramsey allocation is to tax only final goods and not to tax intermediate goods. Thus, the theorem implies that it is optimal not to have intertemporal distortions.

More formally, the result that a Ramsey allocation must be at the boundary implies that there exist supporting prices p and w such that the allocation solves

$$\max\ pC - wN$$

subject to the requirement that the allocation is in the production set.

The first order conditions for this problem include

$$\frac{\mathcal{C}_{ct}}{\mathcal{C}_{ct+1}} = 1 - \delta + F_{k,t+1}, \tag{164}$$

and

$$\frac{\mathcal{N}_{nt}}{\mathcal{N}_{nt+1}} = \frac{F_{n,t}}{F_{n,t+1}} \left(1 - \delta + F_{k,t+1}\right). \tag{165}$$

These are conditions of production efficiency. Condition (164) equates the rates at which c_t is transformed into c_{t+1} through the composite C to the rate at which c_t is transformed into c_{t+1} through capital, k_{t+1}. Condition (165) is the analog for labor in consecutive periods. Notice that, since $\beta^t u_c(t) = U_c \mathcal{C}_{ct}$ and $\beta^t u_n(t) = U_N \mathcal{N}_{nt}$, it follows that $\frac{\mathcal{C}_{ct}}{\mathcal{C}_{ct+1}} = \frac{u_{c,t}}{\beta u_{c,t+1}}$ and $\frac{\mathcal{N}_{nt}}{\mathcal{N}_{nt+1}} = \frac{u_{n,t}}{\beta u_{n,t+1}}$. Thus, conditions (164) and (165) imply that it is optimal to have no intertemporal distortions. They also imply that intratemporal distortions are constant. We summarize this discussion in the following proposition, which is the equivalent to Proposition 4 in the intermediate good economy.

Proposition 4': (No intertemporal distortions ever) Suppose that preferences and technologies are given by (157), (158) and (159) and the wealth restriction must be satisfied. Then, the Ramsey solution has no intertemporal distortions for all $t \geq 0$.

Consider next the case in which the wealth restriction is imposed in units of goods rather than in utility terms. Here, production efficiency is no longer optimal. Nevertheless, it is straightforward to show that an analogue of Proposition 5 holds in the intermediate good economy in the sense that it is not optimal to distort intermediate goods decisions from period one onwards. It is optimal to distort intermediate goods in period zero relative to all other intermediate goods in order to partially tax rents.

The intermediate good economy helps clarify circumstances in which it is optimal not to have intertemporal distortions. We have shown that it is optimal not to have such distortions when the underlying economy can be represented as a constant returns to scale economy in the production of intermediate goods. In this sense we have shown an equivalence between the principle that intermediate goods taxation is undesirable and intertemporal distortions should not be introduced.

In the process of doing so, we have shown that the celebrated result of Atkinson and Stiglitz (1972), that uniform commodity taxation is optimal when preferences are homothetic and separable, follows from the production efficiency result of Diamond and Mirrlees. Thus, our result of no intertemporal distortions is very closely connected to the uniform commodity taxation result.

Production efficiency in an economy without capital How different are the results above from the ones in an economy without capital as in Lucas and Stokey (1983)? In that production economy, uniform taxation is optimal for standard macro preferences. How does that relate to production efficiency? Can we map that economy into an intermediate good economy as we did in the growth model?

In Lucas and Stokey, instead of the intertemporal technology (113), the technology is static and given by

$$c_t + g_t \leq A_t n_t. \tag{166}$$

The map between the original economy and the intermediate good economy is the same except that instead of the capital accumulation firms, we have now *production firms* that use technology (166). The conditions for production efficiency in the intermediate good economy are now

$$\frac{\mathcal{C}_{ct}}{\mathcal{C}_{ct+1}} = \frac{\mathcal{N}_{nt}}{\mathcal{N}_{nt+1}} \frac{A_{t+1}}{A_t}, \tag{167}$$

instead of (164) and (165).

Because $\mathcal{C}_{ct}$ and $\mathcal{N}_{nt}$ are proportionate to $u_{c,t}$ and $u_{n,t}$, condition (167) implies that intratemporal distortions are constant over time. There are no implications for intertemporal wedges because there are no such wedges.

6.1 Production efficiency in an heterogeneous agent economy

Consider now developing an intermediate goods economy that is equivalent to our heterogenous agent economy. Here we think of the intermediate good economy as producing two distinct types of final consumption goods denoted by C^i for $i = 1, 2$. For simplicity, we assume the economy utilizes one common type of final labor, denoted by N. The preferences for household of type i are given by

$$U\left(C^i, N^i\right) = \frac{\left(C^i\right)^{1-\sigma^i} - \frac{1}{1-\beta}}{1-\sigma^i} - \eta\left(N^i\right)^{\psi}. \tag{168}$$

where N^i denotes the amount of the common final labor supplied by type i. The technologies for the capital accumulation and the labor firm are the same as before. Each consumption good is produced by its own constant returns to scale technology given by

$$C^i = \mathcal{C}^i(c_0^i, c_1^i..) = \left[\sum_{t=0}^{\infty} \beta^t \left(c_t^i\right)^{1-\sigma^i}\right]^{\frac{1}{1-\sigma^i}}. \tag{169}$$

for $i = 1, 2$.

The tax system is the same except that we require that the tax rate on final consumption goods must be the same for the two types. We do so to show the equivalence between the intermediate goods economy and the heterogenous agent economy with type independent taxation.

Household of type i maximizes (168) subject to the budget constraint

$$p^i(1+\tau^C)C^i - w(1-\tau^N)N^i \leq (1-l_0)\, V_0^i \tag{170}$$

where p^i denotes the price of final consumption good of type i, in units of the consumption good in period 0.

The consumption firm of type i maximizes

$$p^i C^i - \sum_{t=0}^{\infty} q_t (1+\tau_t^c) c_t^i \tag{171}$$

subject to (169).

The other firms solve the same problems as in the representative agent economy. A competitive equilibrium is defined in the standard fashion.

Next, we show that the competitive equilibria in the intermediate goods economy and the heterogenous agent economy with type independent taxes coincide. A key step in this proof is that the tax rates on the final consumption goods are restricted to be the same. Recall that in the representative agent model we used zero profits for consumption goods producers and replaced for the pre-tax value of consumption in the households budget constraint in the intermediate goods economy by the value of the intermediate goods. Following the same procedure, we can use (161) and write (160) as

$$(1+\tau^C)\sum_{t=0}^{\infty} q_t(1+\tau_t^c)c_t^i - w(1-\tau^N)N^i \leq (1-l_0)\, V_0^i$$

Notice that this budget constraint coincides with the budget constraints in the heterogenous agents economy except for rescaling. Notice that if the tax rates in the two consumption goods were allowed to be different in the intermediate goods economy, the budget constraints would not coincide and the competitive equilibria would be different.

The rest of the argument that the equilibria coincide is the same as in the representative agent economy. Clearly, if we were to set-up an intermediate goods economy that coincided with the heterogenous agent economy with type-dependent taxes, we could allow the tax rates in the two final consumption goods to be different in the intermediate good economy.

Consider now the Ramsey problem in the intermediate goods economy with the same tax rate on both consumption goods. This requirement imposes an additional restriction on the Ramsey problem. Straightforward algebra shows that this constraint can be written as

$$\frac{U^i_{C^i}\mathcal{C}^i_{c_t}}{-U^i_{N^i}\mathcal{N}^i_{n_t}} = \frac{U^j_{C^i}\mathcal{C}^j_{c_t}}{-U^j_{N^i}\mathcal{N}^j_{n_t}},\ t \geq 0. \tag{172}$$

The Ramsey problem is now to maximize utility subject to implementability constraint, the requirement that the allocation is in the production set and (172). The Ramsey problem in the intermediate good economy with type-dependent taxes simply drops (172).

We now have the analog of proposition 13.

Proposition 13': (No intertemporal distortions in heterogenous agent economies) Suppose that preferences for all types of agents are in the class of standard macroeconomic preferences. If all agents have the same preferences, then the Ramsey equilibrium has no intertemporal distortions for all $t \geq 0$. If agents' preferences differ, and if tax rates on consumption and labor income can be type-dependent, then the Ramsey equilibrium has no intertemporal distortions for all $t \geq 0$.

7 Concluding remarks

We characterize cooperative Ramsey allocations in the open economy. We show that free trade is optimal also in the second best Ramsey allocation and that for standard macro preferences capital should never be taxed. We study alternative implementations of the Ramsey allocation including taxes on equity returns, foreign asset returns and firms profits, and value added taxes with and without border adjustments. We discuss the desirability of residence versus source based taxation of asset income and destination versus origin based taxation of goods.

The results on the zero taxation of capital are related to the influential results of Chamley (1986) and Judd (1985) that argue that capital should not be taxed in the steady state but while it should be heavily taxed along a transition. We also relate the results to the more recent literature, in Bassetto and Benhabib (2006) and Straub and Werning (2015), that challenge the optimality of zero taxation of capital in the steady state

To obtain our result that there is no presumption that capital ought to be taxed along the transition, it is important that the initial confiscation be restricted not only directly as is common to assume in the literature, but also indirectly through valuation effects as proposed by Armenter (2008). We discuss the relevance of this assumption and relate it to partial commitment to asset returns.

Another important assumption to shorten the transition of heavy capital taxation is that the tax system may be rich enough, in the sense that no taxes that are available in advanced economies may be left out if relevant for policy. We consider such a rich tax system, but that is not the common assumption in the literature. The assumptions that indirect confiscation is possible, while direct confiscation is not, together with a very restricted tax system, explain the extreme results in Bassetto and Benhabib (2006) and Straub and Werning (2015).

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