Advanced Mathematics

Academic Year: **2016/2017** *Semester:* ***1st Semester***

Instructor(s): Fernando Pacheco

Course Description: This course focuses on dynamic optimization methods and techniques. We start by studying difference and differential equations as a way to represent and analyze dynamic problems. We also study the dynamic systems that come from the solutions to these problems. These problems are then approached as optimization problems, from a calculus of variations or an optimal control perspective.
The course will illustrate how these methods and techniques can be used in various applications, drawing on economic policy examples. However, the focus will remain on gaining a general command of the tools so that they can be applied later in other classes.

Course Content:

1. Difference equations.
   First order linear equations; Stability; Higher order linear equations; systems of equations.
2. Differential equations.
   First order linear equations; Stability; Higher order linear equations; systems of equations; Stability for systems of equations and phase diagrams. (Application: Dornbusch ER determination model and Wilson's extension).
3. Calculus of variations.
   Basic concepts; Notion of functional; Euler-Lagrange conditions; Extensions of the basic problem: constraints, initial conditions and transversality conditions. (Application: shortest distance between two points in a plane)
4. Maximum principle and control theory.
   Basic concepts; The Pontryagin principle; the control formulation and the Hamiltonian function; economic interpretation of the costate variable. (Application: the time-consistency of economic policy)

Course Objectives: Upon successful completion of this course, students should be able to complete the following tasks:
1. Understand the role of time in economic problems;
2. Formulate and/or solve dynamic problems, including phase diagrams (i. e., problems involving systems of dynamic equations);
3. Use the basic tools of dynamic optimization, and in particular be able to solve calculus of variations or optimal control problems (continuous time).

Grading: There will be two written exams, scheduled for the middle and the end of the course. Each exam accounts for half of the final grade. Students with a valid, verifiable reason for

missing the first are allowed to pass only the final exam; those who have missed the exam without a valid, verifiable reason will have a 0 (zero) grade in that exam.
Students are not allowed to consult notes or any other materials during exams.

Extra Costs (case studies, platforms...): Students are expected to make two presentations, based on references [3] to [5] below.

Bibliography:

[1] Alpha C. Chiang, *Fundamental methods in mathematical economics*, 3th edition, Mc-Graw-Hill, 2001.
[2] Morton I. Kamien and Nancy Schwartz, *Dynamic Optimization: the calculus of variations and optimal control in economics and management*, North-Holland, 1981.
[3] Rudiger Dornbusch, *Expectations and Exchange Rate Dynamics*, Journal of Political Economy, vol. 84, n. 6, 1976.
[4] Guillermo Calvo, *On the Time Consistency of Optimal Policy in a Monetary Economy,* Econometrica, vol. 46, n.6, 1978.
[5] David Snower, *Macroeconomic Policy and the Optimal Destruction of Vampires*, Journal of Political Economy, vol. 90, n. 3, 1982.

Biography: Fernando Pacheco has a PhD under the European Doctoral Program in Quantitative Economics (Univ. Catholique de Louvain, London School of Economics and Bonn University). He was Director of the Economics Department and Pro-Rector at UCP. He was also Director-General of the Central Planning Department (public administration), Economic Advisor to the Prime Minister, Secretary of State for Industry and Energy, Secretary of State for the Budget, and Vice President of Iberdrola-Portugal (utility, energy sector). Currently he heads the Technical Unit for the Monitoring of State Owned Enterprises (Ministry of Finance).

Contact(s) and Office hours:
Office hours by appointment.
