Impact of Adverse Selection on Interbank Lending

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Abstract

The question studied here is how do productivity and liquidity shocks interact to impact risk-sharing across financial regions when there is private trading and private information and to determine the role (if any) of the central bank in facilitating risk sharing. Preference shocks create a demand for liquidity, as in Diamond and Dybvig. The fraction of early consumers in a region is stochastic and is private information, known only by the banks in the region. The distribution of the returns to real investments made by the representative regional bank is private information, creating an adverse selection problem in insuring against the shocks. The bank is subject to shocks on both the liability side as well as the asset side of its balance sheet. Private information and private trading interact to amplify the impact of productivity and liquidity shocks, limiting the ability of a region to engage in risk-sharing.

There is an interbank market in which banks from different regions can borrow and lend to smooth liquidity shocks. Banks can insure against idiosyncratic productivity shocks by engaging in securitization, a form of insurance. When there is adverse selection in the securitization process, the composition of a bank's assets between short-term risk free assets and long-term risky assets is distorted, in that high risk regions under-invest in the risk-free asset and low risk regions under-invest in long-term risky assets. The distortion in the composition of assets spills over into the interbank credit market through the interest rate. Acting as a mechanism designer, the central

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bank can implement a private-trading constrained efficient allocation by imposing a restriction on the asset composition of a region, specifically through a liquidity floor.

The question studied here is how do productivity and liquidity shocks interact to impact risk-sharing across financial regions when there is private trading and private information and to determine the role (if any) of the central bank in facilitating risk sharing. The distortion in the composition of assets on a bank's balance created by private information and private trading in the securitization process impacts the interbank market through the interest rate, limiting a regional bank's ability to smooth liquidity shocks.

The distribution of the returns to real investments made by the banks in a region is private information known only to the regional banks. Banks insure against productivity shocks by engaging in securitization of the risky asset. Private information about the distribution of productivity shocks creates an adverse selection problem, which is magnified through private trading arrangements. The private trading arrangements limits the types of restrictions that can be imposed by a central bank, who is acting as a mechanism designer. Securitization and private trading on the asset-side of a regional bank's balance sheet results in a risky region acting as if it were a low-risk region. Risky regions underinvest in the safe asset while low risk regions underinvest in the risky asset, choosing to hold the safe asset as a form of self insurance.

Preference shocks create a demand for liquidity, as in Diamond and Dybvig [4]. The fraction of early consumers in a region is stochastic and is private information, known only by the banks in the region. Since the fraction of impatient households is private information, the opportunities to insure against liquidity shocks is limited. Private trading opportunities impose additional limitations on the types of interventions available to a central bank to provide liquidity insurance. Banks partially smooth liquidity shocks by engaging in borrowing and lending in an interbank market.

The bank is subject to shocks on both the liability side as well as the asset side of its balance sheet. The focus here is how the shocks interact to impact risk-sharing across regions. The composition of the bank's assets affects its ability to borrow in the interbank market to smooth liquidity shocks. As in (FGT), the private trading constrained efficient allocation can be implemented by imposing a liquidity floor - a restriction on the bank's asset portfolio - which then impacts the incentives to securitize the risky assets.

As described by Jacklin [6] in a Diamond-Dybvig model, private trading can severely limit risk sharing. The implications of private trading opportunities for agents' decisions are studied by Farhi, Golosov and Tsyvinski [5] (FGT) in a Diamond-Dybvig banking model and by Labadie [9]-[10] in a Rothschild-Stiglitz adverse selection insurance economy.

1 Basic Model

There are three periods $t = 0, 1, 2$ and a continuum of locations $\phi \in \Phi \equiv [0, 1]$. There is a financial intermediary (a bank) and a representative household at each location. At $t = 0$, all households are endowed with one unit of the consumption good, which is deposited with the bank. The bank invests a portion of this deposit $1 - k$ in a risk-free technology, where one unit invested at $t = 0$ yields one unit of consumption at $t = 1$, and the remainder k is invested in a risky technology yielding a random output at $t = 2$. The risky project cannot be interrupted at $t = 1$.

For each location, the investment k in the risky long-term technology is subject to a productivity shock θ such that $\theta \in \Theta \equiv \{\theta_1, \theta_2\}$, where $0 < \theta_1 < 1 < \theta_2$. The distribution of the productivity shock is independent across locations. The realization θ for each $\phi \in \Phi$ is observed at $t = 2$. At time $t = 0$, each bank observes a signal η, where $\eta \in N \equiv [\underline{\eta}, \bar{\eta}]$, determining the conditional distribution of its productivity shock θ. Define $f(\eta)$ as the probability of observing η. Define $\text{prob}(\theta = \theta_i \mid \eta) = g_i(\eta)$. Let $\bar{\theta}(\eta)$ denote the conditional mean

$$\bar{\theta}(\eta) \equiv g_1(\eta)\theta_1 + g_2(\eta)\theta_2.$$

The realization of η is private information.

Since there is a continuum of banks, a fraction $f(\eta)$ of the banks will realize η at $t = 0$.

Assumption 1 $g_i(\eta) > 0,\ \eta \in N,\ i = \{1, 2\}$.

The following assumption provides a convenient approach for comparing riskiness across regions.

Assumption 2 *Let $g_1(\eta)$ be continuous and increasing in $\eta \in N$ such that $\eta < \hat{\eta}$, then*

$$0 < g_1(\eta) \leq g_1(\hat{\eta}).$$

The type $\hat{\eta}$ region is said to be riskier than the type η region if $\bar{\theta}(\eta) > \bar{\theta}(\hat{\eta})$.

An example such a distribution is to let $N \equiv [\underline{\eta}, \bar{\eta}] \subset (0, 1)$ and set $g(\eta) = \eta$ such that η is uniformly distributed with probability density function $f(\eta) = (\bar{\eta} - \underline{\eta})^{-1}$.

An investment of k at $t = 0$ by a type η bank yields expected output

$$\bar{\theta}(\eta)k^{\alpha}$$

at $t = 2$, where $0 < \alpha < 1$, so the expected return to the investment for a type η bank is

$$E_{\eta}R(\eta) = \frac{\bar{\theta}(\eta)k^{\alpha} - k}{k} = \bar{\theta}(\eta)k^{\alpha-1} - 1.$$

The marginal product of investment in the risk asset $\bar{\theta}(\eta)\alpha k^{\alpha-1}$ approaches infinity as k tends to zero. This implies all regions will choose to invest a positive amount in the long-term risky project, regardless of its riskiness. To ensure every region will invest in both types of technologies the following assumption is made.

Assumption 3

$$\bar{\theta}(\underline{\eta}) \leq \alpha^{-1}$$

This assumption helps to avoid borrowing and lending at $t = 0$ when η is public information.

Households

Households at location ϕ are endowed with one unit of the consumption good, which is deposited with the financial intermediary at location ϕ. A household cares about consumption in periods $t = 1, 2$. At $t = 1$, a household observes a liquidity shock $\upsilon \in \{0, 1\}$. A household has utility

$$\mathcal{U}(c_1, c_2, \upsilon) = (1 - \upsilon)U(c_1) + \upsilon\beta U(c_2),$$

where $0 < \beta < 1$. If $\upsilon = 0$, the household is said to be impatient and, if $\upsilon = 1$, the household is patient.

A fraction h of the households at location ϕ realize $\upsilon = 0$ and the remainder $1 - h$ realize $\upsilon = 1$. All households at location ϕ observe the realization of h for location ϕ. The probability of realizing h is $\pi(h)$ where $h \in H \equiv [\underline{h}, \bar{h}]$. Region ϕ is described by the pair (η, h) and its decisions (l, k) at $t = 0$. Let

$$h^m = \int_H h\pi(h)dh.$$

A region has a high need for liquidity when $h > h^m$ and a low need when $h < h^m$.

Denote $c_1(h, \eta)$ as the time $t = 1$ consumption of an impatient type η household when h is realized. At $t = 0$, the representative type η household at location ϕ has expected utility

$$\int_H \left\{ hU(c_1(h, \eta)) + (1 - h)\beta \sum_i g_i(\eta)U(c_2(\eta, h, \theta_i)) \right\} \pi(h)dh \tag{1}$$

where $0 < \beta < 1$.

Assumption 4 *The function U is continuously differentiable, strictly increasing and strictly concave. As $c \to 0$, $U'(c) \to \infty$ and as $c \to \infty$, $U'(c) \to 0$ (the Inada conditions). Also $U'(c)c$ is monotonic in c.*

A closed-form solution will be derived for the case $U(c) \equiv \ln(c)$. Since the investment cannot be interrupted and $\theta_1 > 0$, there will be no run on financial institutions.

There is no aggregate uncertainty in the model. All risk is idiosyncratic - specifically regional. As will be demonstrated in the derivation of the first-best solution, there is an incentive for the social planner to move resources from high risk to low risk regions at $t = 0$. To simplify the analysis and to make comparison of allocations meaningful, it is assumed resources cannot be transferred across locations at $t = 0$, although there is no problem entering into contingent contracts at $t = 0$.

1.1 Autarky

In autarky, which is defined as the absence of insurance or other forms of trade across locations, a bank at location $\phi \in \Phi \equiv [0, 1]$ realizing η maximizes the expected utility of the representative household in its region (1) with respect to $\{l, k, c_1(\eta, h), c_2(\eta, h, \theta)\}$, subject to the constraints

$$1 = l + k, \tag{2}$$

$$l = hc_1(\eta, h), \tag{3}$$

$$\theta_i k^\alpha = (1 - h)c_2(\eta, h, \theta_i). \tag{4}$$

The first-order conditions with respect to (c_1, c_2, l, k) can be simplified as

$$\int_H U'\left(\frac{1-k}{h}\right)\pi(h)dh = \beta \int_H \sum_i U'\left(\frac{\theta_i k^\alpha}{1-h}\right)\theta_i \alpha k^{\alpha-1}\pi(h)dh \tag{5}$$

The left side is increasing in k and the right side is decreasing. Denote $k_a(\eta)$ as the solution and define $l_a(\eta) \equiv 1 - k_a(\eta)$.

Proposition 1 *Let $\hat{\eta} > \eta$ so $g_1(\hat{\eta}) > g_1(\eta)$. If $U'(c)c$ is decreasing in c, then $k_a(\hat{\eta}) > k_a(\eta)$. If $U'(c)c$ is increasing in c, then $k_a(\hat{\eta}) < k_a(\eta)$ and, if $U'(c)c$ is constant, then $k_a(\eta) = k_a(\hat{\eta})$*

PROOF.

Since $g_2(\eta) = 1 - g_1(\eta)$, the derivative of the right side of (5) with respect to η, holding $k = k_a(\eta)$

fixed, is

$$\frac{\beta\alpha(1-h)}{k}\left[\int_H\left(\frac{\partial g_1(\eta)}{\partial\eta}\right)\left[U'\left(\frac{\theta_1 k^\alpha}{1-h}\right)\frac{\theta_1 k^\alpha}{1-h}-U'\left(\frac{\theta_2 k^\alpha}{1-h}\right)\frac{\theta_2 k^\alpha}{1-h}\right]\pi(h)dh\right]$$

By assumption $\frac{\partial g_1(\eta)}{\partial\eta} > 0$. Since $\frac{\theta_2 k^\alpha}{1-h} > \frac{\theta_1 k^\alpha}{1-h}$, the term in brackets is positive if $U'(c)c$ is decreasing in c and negative if $U'(c)c$ is increasing in c. If $U'(c)c$ is decreasing in c, then as η rises, k must increase to lower the right side of the equation and increase the left side, so $k_a(\hat{\eta}) > k_a(\eta)$. The converse holds when $U'(c)c$ is increasing. If $U'(c)c$ is constant, then the term in brackets is 0. ∎

When $U'(c)c$ is decreasing, agents invest more in the long-term risky asset as a precaution against the low productivity shock. When relative risk aversion is less than one, agents invest less in the risky technology. Consumption in the first period varies inversely with h and second-period consumption varies positively with h and θ. The introduction of storage, which is studied in the appendix, allows partial consumption smoothing against liquidity shocks, partial in the sense storage is strictly nonnegative so liquidity shocks in which a large fraction of the region's population prefers early consumption cannot be smoothed in the absence of borrowing opportunities. The absence of storage creates a demand for liquidity (and a supply) when trade is allowed across regions.[2]

[2] The full problem is stated here: Let $\mu_0(\eta)$ denote the Lagrange multiplier for the first constraint for a type η region; let $\mu_1(\eta, h)$ denote the multiplier for the second, conditional on realizing h, and let $\mu_2(\eta, h, \theta_i)$ denote the multiplier for the constraint in the final period, conditional on realizing θ_i. Since there is no trade, the Inada conditions imply $k > 0$ and $l > 0$, so non-negativity constraints on the investment decisions are unnecessary. The first-order conditions with respect to (c_1, c_2, l, k) are

$$\begin{aligned} hU'(c_1)\pi(h) &= \mu_1(\eta,h)h \\ (1-h)\beta U'(c_2)g_i(\eta)\pi(h) &= \mu_2(\eta,h,\theta_i)(1-h) \\ \mu_0(\eta) &= \int_H \mu_1(\eta,h)dh \end{aligned}$$

The expected intertemporal marginal rate of substitution between $t = 1$ and $t = 2$, equal to

$$\frac{\beta \sum g_i(\eta) U'(\frac{\theta_i k_a^\alpha(\eta)}{1-h})}{U'(\frac{1-k_a(\eta)}{h})},$$

will vary across regions because of different realizations of h and differences in productivity η.

Logarithmic Utility

An analytic solution can be found when $U(c) = \ln(c)$. Define the function $A : H \to [0, 1]$ by

$$A(h) \equiv \frac{\alpha\beta(1-h)}{h + \alpha\beta(1-h)},$$

where $A'(h) < 0$ and $A''(h) > 0$. The investment in the risky asset satisfies

$$k_a \equiv k_a(\eta) = A(h^m),$$

so all regions hold identical portfolios of risk-free and risky assets, despite the differences in productivity. The expected intertemporal marginal rate of substitution

$$\frac{\beta \sum_i g_i(\eta) U'(c_2(\theta_i, h, \eta))}{U'(c_1(\eta, h))} = \frac{\beta h}{1-h} \frac{\sum_i g_i(\eta)\theta_i k_a^\alpha}{1-k_a}$$

varies positively with h.

2 First-Best Solution

All realizations (η, h, θ_i) are publicly observable. A fraction $\int_N g_i(\eta) f(\eta) d\eta$ of regions will realize productivity shock θ_i. A fraction $\pi(h)$ of regions will realize liquidity shock h.

$$\begin{aligned} \mu_0(\eta) &= \int_H \sum_i \mu_2(\eta, h, \theta_i)\theta_i \alpha k^{\alpha-1}\pi(h) dh \\ \mu_1(\eta, h) &= \mu_s(h, \eta) + \sum_i \mu_2(\eta, h, \theta_i) \end{aligned}$$

Eliminating the multipliers μ_0, μ_1, μ_2 results in k solving (5).

A social planner chooses consumption and investment across all locations. Define $\Phi(\eta)$ as the Pareto weight assigned to a type η region such that $\Phi(\eta) > 0$ for $\eta \in N$ and

$$1 = \int_N \Phi(\eta) d\eta.$$

The social planner maximizes

$$W = \int_N \int_H \left[hU(c_1(\eta, h)) + (1-h)\beta \sum_i g_i(\eta) U(c_2(\eta, h, \theta_i)) \right] \Phi(\eta)\pi(h) dh d\eta \tag{6}$$

by choosing $\{k(\eta), l(\eta), c_1(\eta, h), c_2(\eta, h, \theta), s(\eta, h)\}$ subject to the resource constraint for each period

$$1 \geq \int_N [l(\eta) + k(\eta)] f(\eta) d\eta, \tag{7}$$

$$\int_N l(\eta) f(\eta) d\eta \geq \int_N \int_H [hc_1(\eta, h) + s(\eta, h)] f(\eta)\pi(h) d\eta dh, \tag{8}$$

$$\int_N \int_H \left[\sum_i g_i(\eta)\theta_i (k(\eta))^\alpha + s(\eta, h) \right] \pi(h) dh f(\eta) d\eta = \int_N \int_H (1-h) \sum_i g_i(\eta) c_2(\eta, h, \theta_i) f(\eta)\pi(h) d\eta dh$$

and the constraints $s(h, \eta) \geq 0$ and $k(\eta) \leq 1$. Observe $k > 0$ because of diminishing returns. Let λ_t denote the Lagrange multiplier for the resource constraint in period t, $t = 0, 1, 2$, and let λ_s denote the multiplier for the storage constraint and let λ_k denote the multiplier for the constraint $k \leq 1$. The first-order conditions with respect to $(l(\eta), k(\eta), c_1(\eta, h), c_2(\eta, h, \theta), s(\eta, h))$ are

$$\lambda_0 f(\eta) = \lambda_1 f(\eta) \tag{9}$$

$$\lambda_0 f(\eta) + \lambda_k(\eta) = \lambda_2 \sum_i g_i(\eta)\theta_i \alpha (k(\eta))^{\alpha-1} f(\eta) \tag{10}$$

$$\lambda_1 h f(\eta)\pi(h) = \Phi(\eta) h U'(c_1(\eta, h))\pi(h) \tag{11}$$

$$\Phi(\eta)\beta(1-h) g_i(\eta) U'(c_2(\eta, h, \theta))\pi(h) = (1-h) f(\eta)\pi(h) g_i(\eta)\lambda_2 \tag{12}$$

$$\lambda_s(\eta, h) = [\lambda_1 - \lambda_2] f(\eta) \tag{13}$$

It follows from (11) that $c_1(\eta, h)$ is invariant with respect to h and it follows from (12) that $c_2(\eta, h, \theta)$ is invariant with respect to (h, θ). If $\Phi(\eta) = f(\eta)$, then consumption is invariant with respect to η. There are three cases. In the first case, $\lambda_k(\eta) > 0$ and $k(\eta) = 1$. For this to hold, $\alpha\bar{\theta}(\eta) > 1$ so $\lambda_1 > \lambda_2$ and it follows $s(\eta, h) = 0$. This case is ruled out by the assumption $\alpha\bar{\theta}(\eta) < 1$ for all $\eta \in N$. In the second case, $\lambda_k(\eta) = 0$, so $k(\eta) < 1$, and $\lambda_s(\eta) > 0$, so $s(\eta) = 0$ and $\lambda_1 > \lambda_2$. The final case is $\lambda_s(\eta) = 0$ and $\lambda_k(\eta) = 0$, so $k(\eta) < 1$ and $s(\eta) > 0$. This requires $\lambda_1 = \lambda_2$ so $k(\eta) = (\bar{\theta}(\eta)\alpha)^{\frac{1}{1-\alpha}}$. The implication is $\lambda_1 \geq \lambda_2$ for all cases, which creates a lower bound on interest rates. In the discussion below, the nonnegativity constraints will not be imposed and the solution checked to see if these constraints are binding.

$$\lambda_0 = \lambda_1 = \left[\frac{\Phi(\eta)}{f(\eta)}\right] U'(c_1(\eta)), \tag{14}$$

$$\lambda_2 = \left[\frac{\Phi(\eta)}{f(\eta)}\right] \beta U'(c_2(\eta)), \tag{15}$$

$$\lambda_1 = \lambda_2 \alpha \bar{\theta}(\eta)(k(\eta))^{\alpha-1}. \tag{16}$$

Define $G(\cdot)$ as the inverse function of $U'(\cdot)$ so, if $y = U'(c)$ then $c = G(y)$, and observe G is decreasing. Given λ_1, denote the solution to (14) by $\hat{c}_1(\eta, \lambda_1) = G\left(\frac{\lambda_1 f(\eta)}{\Phi(\eta)}\right)$, where $\hat{c}_1$ is decreasing in λ_1.[3] Given λ_2, let $\hat{c}_2(\eta, \lambda_2) = G\left(\frac{\lambda_2 f(\eta)}{\beta\Phi(\eta)}\right)$ denote the solution to (15), with $\hat{c}_2$ decreasing in λ_2. Finally denote the solution to (16) by

$$\hat{k}(\eta, \lambda_1, \lambda_2) = \left(\frac{\lambda_2}{\lambda_1}\bar{\theta}(\eta)\alpha\right)^{\frac{1}{1-\alpha}},$$

and $\hat{l}(\eta, \lambda_1, \lambda_2) = 1 - \hat{k}(\eta, \lambda_1, \lambda_2)$. The function $\hat{k}$ is decreasing in λ_1 and increasing in λ_2. The

[3]The implicit function theorem can be used to show $\hat{c}_1(\eta, \lambda_1)$ is well defined under the assumptions on U. It follows

$$\frac{\partial \hat{c}_1}{\partial \lambda_1} = \frac{f(\eta)}{\Phi(\eta)} \frac{1}{U''(c_1)} < 0.$$

first-best allocation has the standard property

$$\frac{\lambda_2}{\lambda_1} = \frac{\beta U'(c_2(\eta))}{U'(c_1(\eta))} \tag{17}$$

for all $\eta \in N$.

The resource constraints can be written as

$$\int_N \hat{l}(\eta, \lambda_1, \lambda_2) f(\eta) d\eta = \int_N \int_H h\hat{c}_1(\eta, \lambda_1) f(\eta)\pi(\eta) d\eta dh \tag{18}$$

$$\int_N \sum_i g_i(\eta)\theta_i(\hat{k}(\eta, \lambda_1, \lambda_2))^\alpha f(\eta) d\eta = \int_N \int_H (1-h) \sum_i g_i(\eta)\hat{c}_2(\eta, \lambda_2) f(\eta)\pi(h) d\eta dh \tag{19}$$

This forms a system of two equations in two unknowns (λ_1, λ_2). Since $\hat{k}(\eta, \lambda_1, \lambda_2)$ is decreasing in λ_1 and increasing in λ_2, it follows $\hat{l}(\eta, \lambda_1, \lambda_2) \equiv 1 - \hat{k}(\eta, \lambda_1, \lambda_2)$ is increasing in λ_1 and decreasing in λ_2. Given λ_2, the left side of (18) is increasing in λ_1 while the right side is decreasing. Hence there exists a solution $\hat{\lambda}_1(\lambda_2)$, where $\hat{\lambda}_1(\lambda_2)$ is decreasing in λ_2. Substitute $\hat{\lambda_1}(\lambda_2)$ into (19). The right side of (19) is decreasing in λ_2 while the left side is increasing in λ_2 because $\hat{k}(\eta, \lambda_1, \lambda_2)$ is directly increasing in λ_2 while $\hat{k}$ is decreasing in its second argument and $\hat{\lambda_1}(\lambda_2)$ is decreasing in λ_2. Hence a unique solution $(\lambda_1^\star, \lambda_2^\star)$ exists. Define $k_f(\eta) = \hat{k}(\eta, \lambda_1^\star, \lambda_2^\star)$.

The optimal investment function $k_f(\eta)$ has the property low-risk regions invest more than high-risk regions, implying low-risk regions hold fewer short-term risk free investments, regardless of relative risk aversion or how $U'(c)c$ varies with c. While consumption in each period may differ across regions, consumption within a region is invariant with respect to h, or θ_i. Since there is diminishing returns to investment in the risky asset, there is always positive investment in the risky asset even in high-risk regions.

2.0.1 Logarithmic Example

When utility is logarithmic, the first best capital stock in region η satisfies

$$k_f(\eta) = A(h^m)\left[\frac{(\bar{\theta}(\eta))^{\frac{1}{1-\alpha}}}{\int_N (\bar{\theta}(\eta))^{\frac{1}{1-\alpha}} f(\eta)d\eta}\right].$$

It follows

$$\int_N k^f(\eta)f(\eta)d\eta = A(h^m)$$

and $k_f(\eta)$ is decreasing in η so riskier regions invest relatively more in the short-term project. The average investment in the long-term asset is identical with the autarky solution, but there is an optimal distribution over regions. The investment in the risk-free asset in region η is $1 - k_f(\eta)$ and the average investment in the risk-free asset across regions is $1 - A(h^m)$. Consumption in the first-best allocation satisfies

$$\begin{aligned} c_1^\star(\eta, h) &= \frac{1}{h^m + \beta(1-h^m)}\left[1 - k_f(\eta) + \frac{\bar{\theta}(\eta)(k_f(\eta))^\alpha}{r_f}\right] \\ c_2^\star(\eta, h) &= \frac{\beta r^f}{h^m + \beta(1-h^m)}\left[1 - k_f(\eta) + \frac{\bar{\theta}(\eta)(k_f(\eta))^\alpha}{r_f}\right] \end{aligned}$$

Observe in the autarky solution the time $t = 2$ output is

$$\int_N \bar{\theta}(\eta)(k_a)^\alpha f(\eta)d\eta = (A(h^m))^\alpha \bar{\theta}$$

while, in the first-best solution, it is

$$\int_N \bar{\theta}(\eta)(k_f(\eta))^\alpha f(\eta)d\eta = (A(h^m))^\alpha \int_N \bar{\theta}(\eta)\left[\frac{(\bar{\theta}(\eta))^{\frac{1}{1-\alpha}}}{\int_N (\bar{\theta}(\eta))^{\frac{1}{1-\alpha}}}\right]^\alpha f(\eta)d\eta$$

and the difference is

$$(A(h^m))^\alpha\left[\left[\int_N (\bar{\theta}(\eta))^{\frac{1}{1-\alpha}} f(\eta)d\eta\right]^{1-\alpha} - \bar{\theta}\right] > 0.$$

The first best average output in $t = 2$ exceeds that in autarky.[4]

The intertemporal marginal rate of substitution in the first-best allocation is

$$(r_f)^{-1} = \frac{\lambda_2^\star}{\lambda_1^\star} = (A(h^m))^{1-\alpha}(\alpha)^{-1}\left[\int_N (\bar{\theta}(\eta))^{\frac{1}{1-\alpha}} f(\eta)d\eta\right]^{\alpha-1}$$

2.1 First-best competitive equilibria

It is useful to describe how the first-best allocation can be supported as a competitive equilibrium. A contingent claims market structure is described after which the link to the securitization process and interbank lending is discussed. The representative bank in a region has access to contingent claims markets for liquidity shocks and productivity shocks. One unit of consumption delivered at $t = 1$ contingent on realizing h for a type η region can be purchased at a price $p(\eta, h)$. One unit of consumption delivered at $t = 2$ for a type η region with liquidity shock h realizing productivity θ_i can be purchased at $t = 0$ at a price $q_0(\theta_i, \eta, h)$.

The budget constraints of a type η household in $t = 0, 1, 2$ are

$$\begin{aligned} 1 &\geq l + k + \int_H p(\eta, h)x(\eta, h)dh \\ &\quad + \int_H \sum_i q_0(\theta_i, \eta, h)z_0(\theta_i, \eta, h)dh \end{aligned} \tag{20}$$

$$l + x(\eta, h) + \sum_i q_1(\theta_i, h, \eta)z_0(\theta_i, h, \eta) \geq hc_1(\eta, h) + \sum_i q_1(\theta_i, h, \eta)z_1(\theta_i, h, \eta), \tag{21}$$

$$\theta_i k^\alpha + z_1(\theta_i, \eta, h) \geq (1-h)c_2(\eta, h, \theta). \tag{22}$$

At $t = 0$ the household deposits its endowment with the bank, which chooses (k, l) and purchases contingent claims. The $t = 0$ price of one unit of consumption at $t = 1$ contingent on h in region η is $p(\eta, h)$. The price at $t = 0$ of a unit of consumption at $t = 2$ conditional on (η, h, θ_i) is $q_0(\eta, h, \theta_i)$.

[4] Define $\phi(x) = x^{\frac{1}{1-\alpha}}$ and observe ϕ is increasing and convex. It follows from Jensen's Inequality $\phi(Ex) < E(\phi(x))$ or $(Ex)^{\frac{1}{1-\alpha}} < Ex^{\frac{1}{1-\alpha}}$. Moreover $Ex = (Ex)^{\frac{1-\alpha}{1-\alpha}} < (Ex^{\frac{1}{1-\alpha}})^{1-\alpha}$ and the statement follows.

After realizing h at $t = 1$, the price of one unit of consumption at $t = 2$ conditional on (η, h, θ_i) is $q_1(\eta, h, \theta_i)$. At $t = 1$, a realization of h is observed and any contingent claim depending on a value of h other than the realized value becomes worthless. The contingent claims market opens again at $t = 1$ for any bank wishing to adjust its portfolio. Define $Q_f \equiv \{(q_0(\theta_i, \eta, h), q_1(\theta_i, \eta, h)), i = 1, 2, h \in H, \eta \in N\}$. Given η, prices Q_f and the price function $p(h, \eta)$, the bank maximizes the expected utility of households in its region

$$V(\eta, Q, p) = \int_H \left[hU\left(c_1(\eta, h)\right) + (1-h)\beta \sum_i g_i(\eta) U\left(c_2(\eta, h, \theta_i)\right) \right] \pi(h) dh \tag{23}$$

subject to the budget constraints (20)-(22).

The first-order conditions can be simplified as

$$\int_H U'(c_1(\eta, h))\pi(h)dh = \beta \int_H \sum_i g_i(\eta) U'\left(c_2(\eta, h, \theta_i)\right) \theta_i \alpha k^{\alpha-1} \pi(h) dh, \tag{24}$$

$$p(\eta, h) = \frac{U'(c_1(\eta, h))\pi(h)}{\int_H U'(c_1(\eta, h))\pi(h)dh}, \tag{25}$$

$$q_0(\eta, h, \theta_i) = \frac{q_1(\theta_i, h, \eta) U'(c_1(h, \eta))\pi(h)}{\int_H U'(c_1(h, \eta))\pi(h)dh}. \tag{26}$$

$$q_1(\eta, h, \theta_i) = \frac{\beta g_i(\eta) U'(c_2(\eta, h, \theta_i))}{U'(c_1(\eta, h))} \tag{27}$$

Use (25) to express (26) as

$$q_0(\theta_i, h, \eta) = p(h, \eta) q_1(\theta_i, h, \eta)$$

It follows (24) can be expressed as

$$\begin{aligned} \int_H U'(c_1(h, \eta))\pi(h)dh &= \int_H U'(c_1(h, \eta)) \sum_i q_1(\theta_i, h, \eta)\theta_i \alpha k^{\alpha-1} \pi(h) dh \\ &= \int_H p(h, \eta) \left(\int_H U'(c_1(h, \eta))\pi(h)dh \right) \sum q_1(\theta_i, h, \eta)\theta_i \alpha k^{\alpha-1} dh \end{aligned}$$

or

$$k_f(\eta) = \left(\int_H \sum q_0(\theta_i, h, \eta)\theta_i \alpha dh \right)^{\frac{1}{1-\alpha}}. \tag{28}$$

Since the first-best allocation has the property $c_1(\eta, h)$ and $c_2(\eta, h, \theta)$ are invariant with respect to (h, θ), (25) and (26) imply

$$p(\eta, h) = \pi(h) \tag{29}$$

$$q_1(\eta, h, \theta_i) = \frac{\beta g_i(\eta) U'(c_2(\eta))}{U'(c_1(\eta))} \tag{30}$$

Define

$$r_f^{-1} = \frac{\beta U'(c_2(\eta))}{U'(c_1(\eta))} = \frac{\lambda_2^\star}{\lambda_1^\star}$$

where $(\lambda_1^\star, \lambda_2^\star)$ are the solution multipliers for the social planning problem under full information. It follows

$$k_f(\eta) = \left(\frac{\bar{\theta}(\eta)\alpha}{r_f} \right)^{\frac{1}{1-\alpha}}.$$

Solve the budget constraints for z and x and substitute into the $t = 0$ budget constraint to obtain

$$1 - k_f(\eta) + \frac{\bar{\theta}(\eta)(k_f(\eta))^\alpha}{r_f} = \int_H \left[hc_1(\eta) + \frac{(1-h)c_2(\eta)}{r_f} \right] \pi(h)dh \tag{31}$$

The left side is the expected discounted present value of the endowment while the right side is the expected discounted present value of expenditures.

The contingent claims markets described here are related to the securitization process and interbank lending as follows: Securitization is a form of insurance for the asset side of the bank's balance sheet because the risk from productivity shocks is diversified. The bank sells the state contingent payoffs $(\theta_1(k_f(\eta))^\alpha, \theta_2(k_f(\eta))^\alpha)$ occurring with probabilities $(g_1(\eta), g_2(\eta))$ in return for the certain return $\bar{\theta}(\eta)(k_f(\eta))^\alpha$.

To obtain the first-best allocation for liquidity shocks requires liquidity insurance (contingent claims) be available at time $t = 0$. Trading contingent claims at $t = 0$ cannot be replaced with interbank borrowing and lending at $t = 1$. Interbank lending at $t = 1$ allows a type η region to smooth liquidity shocks, conditional on a realization h. If at $t = 1$ banks can borrow and lend at interest rate r, then consumption will satisfy the pair of equations

$$1 - k_f(\eta) + \frac{\bar{\theta}(\eta)(k_f(\eta))^\alpha}{r_f} = \left[hc_1(h,\eta) + \frac{(1-h)c_2(h,\eta)}{r_f}\right] \tag{32}$$

$$r_f^{-1} = \frac{\beta U'(c_2(h,\eta))}{U'(c_1(h,\eta))}. \tag{33}$$

so consumption will vary with h.[5] Hence contingent claims trading of liquidity risk at $t = 0$ is key to implementing the first best allocation as a competitive equilibrium. When the liquidity and productivity shocks are private information, contingent claims can be traded only on the observable shocks θ_i, creating a potential role for a central bank to improve risk sharing, which is discussed later.

3 Private Trading and Private Liquidity Shocks

In a Diamond-Dybvig framework, FGT have shown introducing private trading opportunities changes the incentive to reveal information. If a social planner offers an incentive efficient allocation, agents may choose to consume a different allocation because of trading opportunities.

To understand the implications of private trading in a Diamond Dybvig framework, assume there is no uncertainty about productivity. Investment of k at $t = 0$ in the long term project by a type η bank yields $\bar{\theta}(\eta)k^\alpha$ with certainty at $t = 2$. Location ϕ realizes a liquidity shock h at $t = 1$ and this realization is not observed by any one outside of location ϕ. The assumption h is private

[5]If there is no liquidity insurance and smoothing of liquidity shocks occurs by borrowing and lending r_f may no longer be an equilibrium and $k_f(\eta)$ no longer the optimal investment in the long term asset. This is discussed below.

information implies a contingent claims market to insure the liquidity shock at $t = 0$ will not exist. Regions smooth liquidity shocks by lending and borrowing at $t = 1$ in an interbank market. The model is studied first under the assumption there is no central bank implementing transfers across regions.

The resource constraints for a location η realizing h are

$$1 = k + l \tag{34}$$

$$l = hc_1(\eta, h) + a(\eta, h) \tag{35}$$

$$r_l a(\eta, h) + \bar{\theta}(\eta)k^{\alpha} = (1 - h)c_2(\eta, h), \tag{36}$$

where $a(\eta, h)$ is the amount saved ($a(\eta, h) > 0$) or borrowed ($a(\eta, h) < 0$) by a type η region with liquidity shock h. The interest rate r_l is not a function of (h, η). The first-order conditions can be simplified as[6]

$$r_l^{-1} = \frac{\beta U'(c_2(\eta, h))}{U'(c_1(\eta, h))} \tag{37}$$

$$\int_H U'(c_1(\eta, h))\pi(h)dh = \int_H \beta U'(c_2(\eta, h))\bar{\theta}(\eta)\alpha k^{\alpha-1}\pi(h)dh \tag{38}$$

[6]The first-order conditions with respect to l, k, c_1, c_2, a are

$$\mu_0(\eta) = \int_H \mu_1(\eta, h)dh$$

$$\mu_0(\eta) = \int_H \mu_2(\eta, h)\bar{\theta}\alpha k^{\alpha-1}dh$$

$$\mu_1(\eta, h)h = hU'(c_1(\eta, h))\pi(h)$$

$$\mu_2(\eta, h)(1 - h) = \beta(1 - h)U'(c_2(\eta, h))\pi(h)$$

$$\mu_1(\eta, h) = \mu_2(\eta, h)r_l$$

It follows

$$
\begin{aligned}
0 &= \int_H \frac{U'(c_1(\eta,h))}{r_l}\bar{\theta}(\eta)\alpha k^{\alpha-1}\pi(h)dh - \int_H U'(c_1(\eta,h))\pi(h)dh \\
&= \int_H \beta U'(c_2(\eta,h))[\bar{\theta}(\eta)\alpha k^{\alpha-1} - r_l]\pi(h)dh,
\end{aligned}
$$

which can be further simplified as

$$
0 = \left[\beta \int_H U'(c_2(\eta,h))\pi(h)dh\right][\bar{\theta}(\eta)\alpha k^{\alpha-1} - r_l]. \tag{39}
$$

Given r_l, solving for k results in

$$
k_l(\eta, r_l) \equiv \left(\frac{\bar{\theta}(\eta)\alpha}{r_l}\right)^{\frac{1}{1-\alpha}}. \tag{40}
$$

The market-clearing conditions are

$$
\begin{aligned}
\int_N l(\eta)f(\eta)d\eta &= \int_N \int_H hc_1(\eta,h)\pi(h)f(\eta)dhd\eta && (41)\\
\int_N \bar{\theta}(\eta)(k_l(\eta,r_l))^{\alpha}f(\eta)d\eta &= \int_N \int_H (1-h)c_2(\eta,h)f(\eta)\pi(h)d\eta dh && (42)\\
0 &= \int_N \int_H a(\eta,h)\pi(h)f(\eta)dhd\eta && (43)
\end{aligned}
$$

Define

$$
W(k,r,\eta) \equiv 1 - k + \frac{\bar{\theta}(\eta)k^{\alpha}}{r_l}.
$$

The budget constraint at $t = 2$ can be solved for $a(\eta, h)$ and substituted into the $t = 1$ budget set so

$$
W(k_l(\eta,r_l), r_l, \eta) = \left[hc_1(\eta,h) + \frac{(1-h)c_2(\eta,h)}{r_l}\right]
$$

A type η region can borrow or lend in the interbank market to smooth liquidity shocks but c_1 and c_2 will vary with respect to h.

3.1 Logarithmic Utility

Define

$$\tau(h) \equiv \frac{(1-h)\beta}{h+(1-h)\beta}$$

The equilibrium interest rate when utility is logarithmic satisfies

$$\frac{1}{r_l} = \alpha^{-1}\left\{\frac{[\int_H \tau(h)\pi(h)dh]^{1-\alpha}}{\left[\int_N (\bar{\theta}(\eta))^{\frac{1}{1-\alpha}} f(\eta)d\eta\right]^{1-\alpha}\{\int_H[\tau(h)+(1-\tau(h))\alpha^{-1}]\pi(h)dh\}^{1-\alpha}}\right\}$$

The equilibrium consumption satisfies

$$\begin{aligned} h\hat{c}_1(\eta, h, r_l, W) &= (1-\tau(h))W(k_l(\eta, r_l), r_l, \eta) \\ (1-h)\hat{c}_2(,\eta, h) &= r_l\tau(h)W(k_l(\eta, r_l), r_l, \eta) \end{aligned}$$

Proposition 2 *For logarithmic utility $r_l < r_f$.*

Proof.

The functions A and τ are decreasing and convex in h. It follows from Jensen's Inequality $A(h^m) < \int_H A(h)\pi(h)dh$. Observe

$$A(h) = \frac{\tau(h)}{\alpha^{-1}(1-\tau(h))+\tau(h)}$$

Define $\phi(\tau)$ by

$$\phi(\tau) \equiv \frac{\tau}{\alpha^{-1}(1-\tau)+\tau}$$

and observe $\phi(\tau)$ is increasing in τ. From the convexity of τ,$\tau(h^m) < \int_H \tau(h)\pi(h)dh$. It follows

$$\phi(\tau(h^m)) < \phi\left(\int \tau(h)\pi(h)dh\right)$$

Hence

$$(A(h^m)) - \left[\frac{\int_H \tau(h)\pi(h)dh}{\int_H\left[\tau(h)+\frac{1}{\alpha}(1-\tau(h))\right]\pi(h)dh}\right] < A(h^m)) - \phi(\tau(h^m)) = A(h^m) - A(h^m) = 0$$

so

$$\frac{1}{r_f} - \frac{1}{r_l} = \frac{1}{\alpha}\left[\frac{1}{\int_N (\bar{\theta}(\eta))^{\frac{1}{1-\alpha}}}\right]^{1-\alpha}\left[(A(h^m))^{1-\alpha} - \left[\frac{\int_H \tau(h)\pi(h)dh}{\int_H \left[\tau(h) + \frac{1}{\alpha}(1-\tau(h))\right]\pi(h)dh}\right]^{1-\alpha}\right] < 0$$

∎

The lower interest rate in the model with a liquidity shock and private exchange results in a higher level of investment in the risky asset for each region, or

$$k_l(\eta, r_l) > k_f(\eta)$$

and in this sense, there is underinvestment in the safe, short-term asset.

3.1.1 Private Trading Constrained Efficient Allocations

Bhattacharya and Gale [2] derive the second best (incentive efficient) allocation in a framework similar to the one studied here. In their model the central bank is a mechanism designer and implements incentive compatible transfers across regions. The key assumption to implementing these transfers is households in each region must consume the allocation specified by the central bank. When regions can engage in trading in the form of borrowing and lending, regions may choose to consume an allocation different from the one specified by the central bank because the incentive to reveal information have been changed. (FGT) examine the impact of private trading, in the form of borrowing and lending, on the incentive to reveal information in a standard Diamond-Dybvig model.

The social planner (central bank) chooses a feasible allocation $\{c_1(h,\eta), c_2(h,\eta)\}_{h\in H, \eta \in N}$ and an investment portfolio $\{1-k(\eta), k(\eta)\}_{\eta\in N}$. While η is observable by the central bank, h is not. A region realizing $h \in H$ may announce a $\hat{h} \neq h$ if it chooses. A type η region realizing h solves

$$\hat{V}(\{c_1(h,\eta), c_2(h,\eta)\}, h, r_l, \eta) = \max_{\{\hat{h}, x_1, x_2\}} [hU(x_1) + \beta(1-h)U(x_2)] \tag{44}$$

subject to $\hat{h} \in H$ and

$$\hat{h}c_1(\hat{h}, \eta) + \frac{(1-\hat{h})c_2(\hat{h}, \eta)}{r_l} \geq hx_1 + (1-h)\frac{x_2}{r_l}, \tag{45}$$

where all agents take the interest rate r_l and menu $\{c_1(h, \eta), c_2(h, \eta)\}_{h \in H, \eta \in N}$ as given. As in FGT, all type η regions will announce the $\hat{h}$ solving

$$\max_{\{\hat{h}\}} \left[\hat{h}c_1(\hat{h}, \eta) + \frac{(1-\hat{h})c_2(\hat{h}, \eta)}{r}\right]$$

Denote this value as $I(\eta)$. Given $I(\eta)$, the (η, h) region solves (44), picking x_1, x_2 to satisfy

$$\frac{1}{r_l} = \frac{\beta U'(x_2)}{U'(x_1)} \tag{46}$$

$$I(\eta) = hx_1 + \frac{(1-h)x_2}{r_l} \tag{47}$$

Denote the solution as $\hat{x}(\eta, h, I(\eta), r_l) = \{\hat{x}_1(\eta, h, I(\eta), r_l), \hat{x}_2(\eta, h, I(\eta), r_l)\}$.

The central bank maximizes (6) by choosing $\{c_1(\eta, h), c_2(\eta, h)\}_{\{\eta \in N, h \in H\}}$ and $\{1-k(\eta), k(\eta)\}_{\eta \in N}$ subject to the resource constraints and the incentive compatibility constraints

$$\int_N (1-k(\eta))f(\eta)d\eta = \int_N \int_H hc_1(\eta, h)f(\eta)\pi(h)d\eta dh \tag{48}$$

$$\int_N \bar{\theta}(\eta)(k(\eta))^\alpha f(\eta)d\eta = \int_N \int_H (1-h)c_2(\eta, h)f(\eta)\pi(h)d\eta dh \tag{49}$$

$$hU(c_1(\eta, h)) + (1-h)\beta U(c_2(\eta, h)) \geq \hat{V}(\{c_1(h, \eta), c_2(h, \eta)\}, h, r_l, \eta) \tag{50}$$

The difference between this specification and the standard problem is the form of the incentive compatibility constraint. The right side of (50) is the indirect utility function of a type (η, h) region because the opportunity to borrow and lend in $t = 1$ changes the incentive to reveal information.

Using the approach described in FGT, the problem can be restated in a more convenient form. First, let

$$V(\eta, I(\eta), r_l, h) = hU(\hat{x}_1(\eta, h, I(\eta), r_l)) + (1-h)\beta U(\hat{x}_1(\eta, h, I(\eta), r_l))$$

FGT demonstrate

$$V(\eta, I(\eta), r_l, h) = \hat{V}(\{c_1(h,\eta), c_2(h,\eta)\}, h, r_l, \eta).$$

where I was defined earlier. The social planning problem can be restated as

$$\max_{\{I(\eta),k(\eta),r_l\}} \left[\int_N \int_H V(\eta, I(\eta), r_l, h) f(\eta)\pi(h) d\eta dh\right] \tag{51}$$

subject to

$$\int_N (1-k(\eta)) f(\eta) d\eta = \int_N \int_H h\hat{x}_1(\eta, h, I(\eta), r_l) f(\eta)\pi(h) d\eta dh \tag{52}$$

$$\int_N \bar{\theta}(\eta)(k(\eta))^{\alpha} f(\eta) d\eta = \int_N \int_H (1-h)\hat{x}_1(\eta, h, I(\eta), r_l) f(\eta)\pi(h) d\eta dh \tag{53}$$

By construction, the incentive compatibility constraints are satisfied because all type η regions receive the identical level of resources in discounted present value $I(\eta)$ and face the same interest rate r_l. Let λ_t for $t = 1, 2$ denote the multipliers. The first order condition for $k(\eta)$ is

$$\lambda_1 = \lambda_2 \bar{\theta}(\eta) \alpha k^{\alpha-1}$$

or

$$\hat{k}\left(\eta, \frac{\lambda_2}{\lambda_1}\right) = \left[\frac{\lambda_2}{\lambda_1}\bar{\theta}(\eta)\alpha\right]^{\frac{1}{1-\alpha}}$$

Observe the first-best investment allocation $\{1-k_f(\eta), k_f(\eta)\}$ satisfies this condition. For a given allocation $\hat{k}(\eta, \frac{\lambda_2}{\lambda_1})$, the optimal $(I(\eta), r_l)$ is the solution to

$$\int_N (1-\hat{k}\left(\eta, \frac{\lambda_2}{\lambda_1}\right) f(\eta) d\eta = \int_N \int_H h\hat{x}_1(\eta, h, I(\eta), r_l) f(\eta)\pi(h) d\eta dh \tag{54}$$

$$\int_N \bar{\theta}(\eta)\left(\hat{k}\left(\eta, \frac{\lambda_2}{\lambda_1}\right)\right)^{\alpha} f(\eta) d\eta = \int_N \int_H (1-h)\hat{x}_1(\eta, h, I(\eta), r_l) f(\eta)\pi(h) d\eta dh \tag{55}$$

The private trading incentive constrained allocation can be supported as a competitive equilibrium when

$$I(\eta) = (1-\hat{k}\left(\eta, \frac{\lambda_2}{\lambda_1}\right) + \frac{\bar{\theta}(\eta)\left(\hat{k}\left(\eta, \frac{\lambda_2}{\lambda_1}\right)\right)^{\alpha}}{r_l}$$

Substitute this function into (54)-(55), resulting in a system of two equations in two unknowns $\left(\frac{\lambda_1}{\lambda_2}, r_l\right)$. As in the paper by FGT, the solution has the property

$$\left(\frac{\lambda_1}{\lambda_2}\right) \neq r_l$$

The social planner will choose the portfolio allocation $(k_f(\eta))$ such that $\frac{\lambda_1}{\lambda_2} = \bar{\theta}(\eta)\alpha(k(\eta))^{\alpha-1}$ attained in the first-best allocation. The intertemporal marginal rate of substitution satisfies $\frac{1}{r_l} = \frac{(1-h)\beta U'(c_2(h,\eta))}{hU'(c_1(\eta,h))}$.

4 Private Information and Securitization

When there is private information, an incentive-efficient allocation, which is constrained efficient (second best), is generally difficult to decentralize, in part because the set of incentive compatible allocations is typically not convex, creating a consumption externality. A complete discussion for a Rothschild-Stiglitz insurance economy is in Prescott and Townsend [10] and Bisin and Gottardi [4].[7] In these settings, a contract is a bundle of contingent claims priced in such a way an agent self-selects into the appropriate market. Agents have an incentive to unbundle the contingent claims in an insurance contract to eliminate arbitrage profits. When private trading cannot be prevented, these arbitrage profit opportunities will be eliminated.

To explain the implications of private trading, assume a region's only uncertainty is the productivity shock θ, so the fraction of early and late consumers is deterministic and equal across regions. In the full information competitive equilibrium, a type η region trades at prices $Q^p(\eta) \equiv (q(\theta_1, \eta), q(\theta_2, \eta))$. If the portfolio of contingent claims held by a type η bank is $(z(\theta_1, \eta), z(\theta_2, \eta))$,

[7] As demonstrated by Bisin and Gottardi, an incentive-efficient allocation can be decentralized in many adverse selection settings if contracts are exclusive, markets are separated by type, and private-trading arrangements are prevented, so a constrained version of the Second Welfare Theorem holds.

the value of that portfolio will differ across markets separated by type η. In other words, if $\hat{\eta} \neq \eta$, where $\hat{\eta}, \eta \in N$, then

$$\sum_i q(\theta_i, \hat{\eta}) z(\theta_i, \eta) \neq \sum_i q(\theta_i, \eta) z(\theta_i, \eta)$$

and an arbitrage profit opportunity exists. For a given portfolio (z_1, z_2), a type η bank will trade in the $\hat{\eta} \in N$ market yielding the highest profit. The result of all banks engaging in the elimination of arbitrage profits will result in prices being equalized across markets, or

$$q(\theta_i, \eta) = q(\theta_i),$$

which is discussed at length in Labadie [10].

If the other regions can observe the portfolio (l, k) selected by region η, then it would seem the region's type could be inferred from its portfolio. As will be demonstrated later, because all regions facing identical prices in the contingent claims markets and all regions have identical initial endowment, all regions will select the same portfolio, regardless of type.

The fraction of patient and impatient households in a location is assumed to be constant h^m. Regions differ in terms of the conditional mean $\bar{\theta}(\eta)$ of the productivity shock, but η is now private information. Contingent claims based on variables that are verifiable, such as the realization θ_i, are traded, but markets by type η cannot be separated so all banks face the same prices $q(\theta_i)$, $i = 1, 2$ in trading at time $t = 0$. Since no new information is revealed between periods 0 and 1, the contingent claims market can reopen at $t = 1$, but no bank will choose to trade in it. The budget constraints for periods $t = 0, 1, 2$ are

$$1 = l + k + \sum_i q(\theta_i) z_0(\theta_i, \eta), \tag{56}$$

$$l = h^m c_1(\eta), \tag{57}$$

$$\theta_i k^\alpha + z_0(\theta_i, \eta) = (1 - h^m) c_2(\eta, \theta_i). \tag{58}$$

Let $\mu_0(\eta), \mu_1(\eta), \mu_2(\theta_i, \eta)$ denote the multipliers for each constraint. The first-order conditions with respect to (l, k, c_1, c_2, z_0) are

$$\mu_1(\eta) = \mu_0(\eta) \tag{59}$$

$$\mu_0(\eta) = \sum_i \mu_2(\theta_i, \eta)\theta_i \alpha k^{\alpha-1} \tag{60}$$

$$h\mu_1(\eta) = hU'(c_1(\eta)) \tag{61}$$

$$(1-h)\mu_2(\theta_i, \eta) = (1-h)\beta g_i(\eta)U'(c_2(\eta, \theta_i)) \tag{62}$$

$$\mu_0(\eta)q(\theta_i) = \mu_2(\eta, \theta_i) \tag{63}$$

Use (63) to express (60) as

$$\begin{aligned} 1 &= \sum_i \left(\frac{\mu_2(\eta, \theta_i)}{\mu_0(\eta)} \right) \theta_i \alpha k^{\alpha-1} \\ &= \sum_i q(\theta_i)\theta_i \alpha k^{\alpha-1}. \end{aligned}$$

Let $Q_p = (q(\theta_1), q(\theta_2))$. Solve this equation for k,

$$k_p(Q_p) = \left[\sum_i q(\theta_i)\theta_i \alpha \right]^{\frac{1}{1-\alpha}}. \tag{64}$$

Notice (64) does not depend on the region's realization of η. As a result, all banks invest the same amount in the long term project. To understand this result, solve (58) for $z_0(\eta, \theta_i)$ and substitute into (57), which is then substituted into (56) and simplified to obtain

$$1 - k_p(Q_p) + \sum_i q(\theta_i)\theta_i (k_p(Q_p))^\alpha = h^m c_1(\eta) + \sum_i q(\theta_i)(1-h^m)c_2(\eta, \theta_i). \tag{65}$$

Since the prices are not functions of the unobservable η, the left-side, which is the bank's discounted present value of resources, does not depend on η.

It follows a bank in region η will choose $(c_1(\eta), c_2(\eta, \theta_i))$ to satisfy

$$q(\theta_1) = \frac{\beta g_1(\eta) U'(c_2(\eta, \theta_1))}{U'(c_1(\eta))}, \tag{66}$$

$$q(\theta_2) = \frac{\beta g_2(\eta) U'(c_2(\eta, \theta_2))}{U'(c_1(\eta))}, \tag{67}$$

and (65). Given Q^p, this is a system of three equations in three unknowns $c_1(\eta), c_2(\eta, \theta_i)$, $i = 1, 2$. For a given $c_1(\eta)$ and Q^p, solve (66) and (67) for $c_2(\eta, \theta_i)$

$$c_2(\eta, \theta_i, c_1, Q^p) \equiv G\left(\frac{q(\theta_i) U'(c_1)}{\beta g_i(\eta)}\right).$$

Substitute these functions into (65) and solve for c_1,

$$1 - k_p(Q^p) + \sum_i q(\theta_i)\theta_i (k_p(Q^p))^\alpha = h^m c_1 + \sum_i q(\theta_i)(1 - h^m)\hat{c}_2(\eta, \theta_i, c_1, Q^p) \tag{68}$$

Observe $G\left(\frac{q(\theta_i)U'(c_1)}{\beta g_i(\eta)}\right)$ is increasing in c_1, so the right side is increasing in c_1, while the left side is constant. Denote the solution by $\hat{c}_1(\eta, Q^p)$. Also define $\hat{c}_2(\eta, \theta_i, Q^p) \equiv c_2(\eta, \theta_i, \hat{c}_1(\eta, Q^p), Q^p)$.

The market-clearing conditions with these functions substituted in are

$$1 - k_p(Q^p) = \int_N h^m \hat{c}_1(\eta, Q^p) f(\eta) d\eta \tag{69}$$

$$\int_N \bar{\theta}(\eta) f(\eta) (k_p(Q^p))^\alpha d\eta = \int_N (1 - h^m) \sum_i g_i(\eta) \hat{c}_2(\eta, \theta_i, Q^p) f(\eta) d\eta \tag{70}$$

This is a system of two equations in the two unknowns $q(\theta_1), q(\theta_2)$. Observe $k_p(Q^p)$ is increasing in $q(\theta_1)$ and $q(\theta_2)$ so the left side of (69) is decreasing in the prices.

The assumption private trading cannot be prevented when there is adverse selection has the important implication the distribution of investment over the different regions will be inefficient, relative to the first best. All regions, regardless of productivity risk, will invest the same amount in the long-term project. Hence, high risk regions will over-invest in the long-term project and low risk firms, unable to secure actuarially fair insurance for the low productivity shock θ_1, will over-invest in the short term riskless project. Low risk regions in this setting are regions for which $\bar{\theta}(\eta) > q(\theta_1)\theta_1 + q(\theta_2)\theta_2$ and high risk regions satisfy $\bar{\theta}(\eta) < q(\theta_1)\theta_1 + q(\theta_2)\theta_2$.

Observe the second period consumption $c_2(\eta, \theta_i)$ will vary with θ_i. Since the insurance offered through the contingent claims market is not priced fairly, meaning $q(\theta_i)$ is not proportional to $g_i(\eta)$, the regions will optimally choose to take on risk, under or over insuring against the low productivity shock θ_1. Whether total capital $k_p(Q^p)$ is above or below total capital in the first best allocation will depend on the curvature of the utility function.

4.0.2 Example with Logarithmic Utility

When utility is logarithmic

$$k_p(Q^p) = \left(\sum q(\theta_i)\theta_i\alpha\right)^{\frac{1}{1-\alpha}}$$

and

$$c_1(\eta, Q^p) = \frac{1 - k_p(Q^p) + \sum q_i\theta_i(k_p(Q^p)^\alpha}{h^m + (1-h^m)\beta}$$

and

$$c_{2i}(\eta) = \frac{\beta g_i(\eta)c_1(\eta)}{q(\theta_i)}$$

The resource constraint for $t = 1$ is

$$1 - k_p(Q^p) = h^m \left[\frac{1 - k_p(Q^p) + \sum q_i\theta_i(k_p(Q^p))^\alpha}{h^m + (1-h^m)\beta}\right]$$

Substitute for $k^p(Q^p)$ and rewrite to show

$$\sum_i q(\theta_i)\theta_i = (A(h^m))^{1-\alpha}\alpha^{-1}$$

and define

$$H \equiv (A(h^m))^{1-\alpha}\alpha^{-1}.$$

Define

$$\psi(q(\theta_1), q(\theta_2)) \equiv \int_N \sum_i \left[\frac{g_i^2(\eta)}{q(\theta_i)}\right] f(\eta) d\eta.$$

The second-period resource constraint can be expressed as

$$\psi(q(\theta_1), q(\theta_2)) = \Lambda_p \equiv \left[\frac{\alpha^{\frac{\alpha}{1-\alpha}} H^{\frac{1}{1-\alpha}} (h^m + \beta(1-h^m)\bar{\theta} H^{-1}}{\beta(1-h^m)(1+\alpha^{\frac{\alpha}{1-\alpha}} H^{\frac{1}{1-\alpha}}}\right] \tag{71}$$

Since $q(\theta_1)\theta_1 + q(\theta_2)\theta_2 = H$, $q(\theta_1) = \frac{1}{\theta_1}[H - q(\theta_2)\theta_2]$. Substitute this into $\psi(q(\theta_1, q(\theta_2))$ and solve for q_2

$$0 = q_2^2 \Lambda \theta_2 + q_2 \left[\int_N [g_1^2(\eta)\theta_1 - g_2^2(\eta)\theta_2] f(\eta) d\eta\right] + \int_N g_2^2(\eta) H f(\eta) d\eta$$

which is quadratic in q_2.

The key result is the distribution of the risky investment is degenerative as a result of the distorted securitization process.

5 Private Information and Interbank Lending

In the economy of Section 3, the central bank can implement transfers across different regions to help smooth out liquidity shocks because η is known. In this section, both η and h are private information so these type-based transfers are not feasible. It will be demonstrated a region's investment portfolio will not reveal information about its type. Since h cannot be observed by outsiders, there will not be a contingent claims market for smoothing liquidity shocks. There is an interbank market in which banks can borrow and lend at the interest rate r at time $t = 1$. The market for claims contingent on the productivity shock is now open at both time $t = 0, 1$ because banks observe h at time $t = 1$ and may wish to adjust their portfolio. A type η bank faces constraints

$$1 = k + l + \sum_i q_0(\theta_i) z_0(\eta, \theta_i) \tag{72}$$

$$l + \sum_i q_1(\theta_i) z_0(\eta, \theta_i) = hc_1(\eta, h) + a(\eta, h) + \sum_i q_1(\theta_i) z_1(\eta, \theta_i, h) \tag{73}$$

$$ra(\eta, h) + z_1(\eta, \theta_i) + \theta_i k^\alpha = (1-h) c_2(\eta, h, \theta_i). \tag{74}$$

The first-order conditions with respect to $l, k, c_1, c_2, z_0, z_1, a$ are

$$\mu_0(\eta) = \int_H \mu_1(\eta, h) dh \tag{75}$$

$$\mu_0(\eta) = \int_H \sum_i \mu_2(\eta, h, \theta_i) \theta_i \alpha k^{\alpha-1} dh \tag{76}$$

$$\mu_1(\eta, h) h = hU'(c_1(\eta, h)) \pi(h) \tag{77}$$

$$\mu_2(\eta, h, \theta_i)(1-h) = \beta g_i(\eta)(1-h) U'(c_2(\eta, h, \theta_i)) \pi(h) \tag{78}$$

$$\mu_0(\eta) q_0(\theta_i) = \int_H q_1(\theta_i) \mu_1(\eta, h) dh \tag{79}$$

$$\mu_1(\eta, h) q_1(\theta_i) = \mu_2(\eta, h, \theta_i) \tag{80}$$

$$\mu_1(\eta, h) = r \sum_i \mu_2(\eta, h, \theta_i) \tag{81}$$

The multipliers can be eliminated using (77) and (78). It follows from (75) and (76) that (76) can be expressed as

$$\int_H U'(c_1(\eta, h)) \pi(h) dh = \beta \int_H \sum_i g_i(\eta) U'(c_2(\eta, h, \theta_i)) \theta_i \alpha k^{\alpha-1} \pi(h) dh. \tag{82}$$

Now (**??**) can be written as

$$q_1(\theta_i) = \frac{\beta g_i(\eta) U'(c_2(\eta, h, \theta_i))}{U'(c_1(\eta, h)}, \tag{83}$$

so (82) is

$$\begin{aligned} \int_H U'(c_1(\eta, h)) \pi(h) dh &= \beta \int_H \sum_i \frac{g_i(\eta) U'(c_2(\eta, h, \theta_i))}{U'(c_1(\eta, h))} U'(c_1(\eta, h)) \theta_i \alpha k^{\alpha-1} \pi(h) dh \\ &= \int_H U'(c_1(\eta, h)) \sum_i q_1(\theta_i) \theta_i \alpha k^{\alpha-1} \pi(h) dh \end{aligned}$$

or

$$1 = \sum_i q_1(\theta_i)\theta_i \alpha k^{\alpha-1} \tag{84}$$

Equation (79) satisfies

$$q_0(\theta_i)\int_H U'(c_1(\eta,h))\pi(h)dh = q_1(\theta_i)\int_H U'(c_1(\eta,h))\pi(h)dh$$

or $q(\theta_i) \equiv q_0(\theta_i) = q_1(\theta_i)$. Hence the equilibrium level of capital is

$$k_b(q(\theta_1), q(\theta_2)) \equiv \left(\alpha \sum_i q(\theta_i)\theta_i\right)^{\frac{1}{1-\alpha}} \tag{85}$$

It follows from(80)(81)

$$r_b^{-1} = \sum_i q(\theta_1) = \beta \frac{\sum g_i(\eta)U'(c_2(\eta,h,\theta_i))}{U'(c_1(\eta,h))} \tag{86}$$

The type η bank chooses consumption $c_1(h,\eta)$ to satisfy

$$1-k_b((q(\theta_1),q(\theta_2))-\sum q(\theta_i)\theta_i(k_b(q(\theta_1),q(\theta_2)))^\alpha = hc_1(h,\eta)+(1-h)\sum q(\theta_i)G\left(\frac{q(\theta_i)U'(c_1(h,\eta))}{\beta g_i(\eta)}\right). \tag{87}$$

The market-clearing conditions are

$$\begin{aligned} 1 - k_b(q(\theta_1), q(\theta_2)) &= \int_H \int_N (1-h)c_1(h,\eta,q_1,q_2)f(\eta)\pi(h)d\eta dh \\ \bar{\theta}(k_b(q(\theta_1), q(\theta_2)))^\alpha &= \int_H \int_N (1-h)\sum_i g_i(\eta)c_2(\theta_i,h,\eta,q_1,q_2)f(\eta)\pi(h)d\eta dh \end{aligned}$$

6 Conclusion

Private information plays an important role in financial arrangements and pricing. The assumption contractual arrangements cannot be made exclusive leads to private trading, altering the incentives to reveal information. The question addressed here is how private information and private trading interact to impact a regional bank's investment portfolio, securitization of that portfolio, and the ability to smooth liquidity shocks in the interbank market.

A regional bank observes a random variable determining the distribution of its risky investments. It also observes a random variable determining the fraction of households in its region wishing for early consumption. The interaction of private information and private trading creates an incentive for all regions to act as if they are low risk in the securitization process, resulting in an overinvestment in the risky asset by high risk regions and an overinvestment in the safe asset by low risk regions. This distortion in the composition of assets impacts the region's ability to smooth liquidity shocks. As in FGT, the private-trading constrained efficient allocation can be implemented by imposing a liquidity floor, a restriction on the composition of the investment portfolio, which impacts the equilibrium interest rate in the interbank market.

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